Complex Numbers Fundamentals

Complex Numbers Need in AC Circuit analysis

- Most often in electronics and under AC condition, analysis involving capacitors and inductors in RC-LR-LRC circuits requires the use of complex number arithmetic.

- Capacitors and inductors under AC exhibit ohmic characteristics and are denoted by the capacitive reactance and inductive reactance.

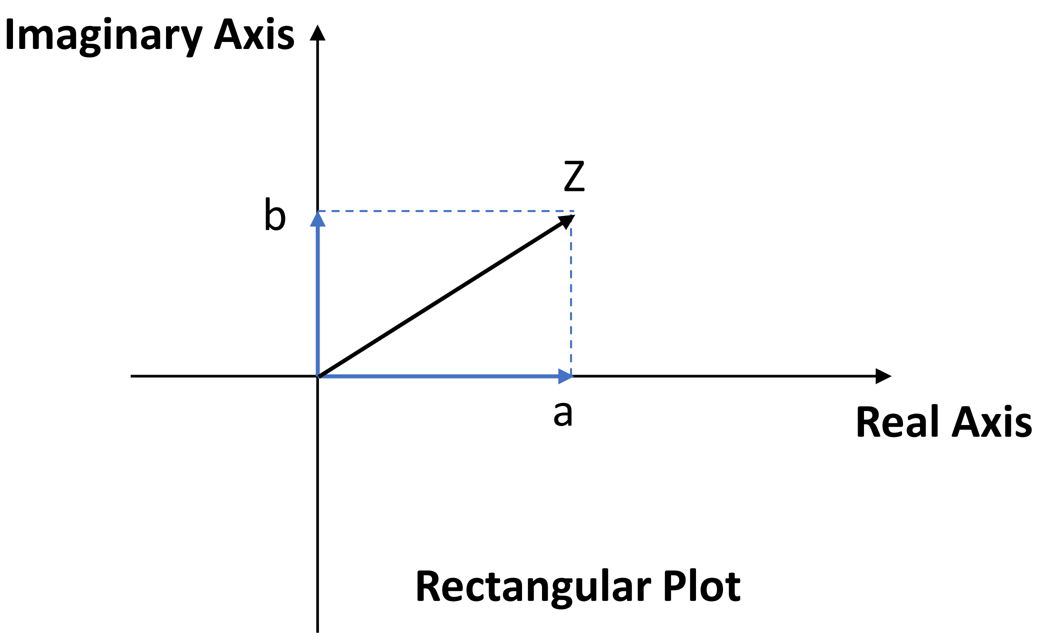
- Both passive elements reactances are represented as imaginary numbers where the capacitive reactance is a negative imaginary number $-jX_c$, and the inductive reactance is a positive imaginary number jX_L .

- The resistance on the other hand is represented as a real number R.

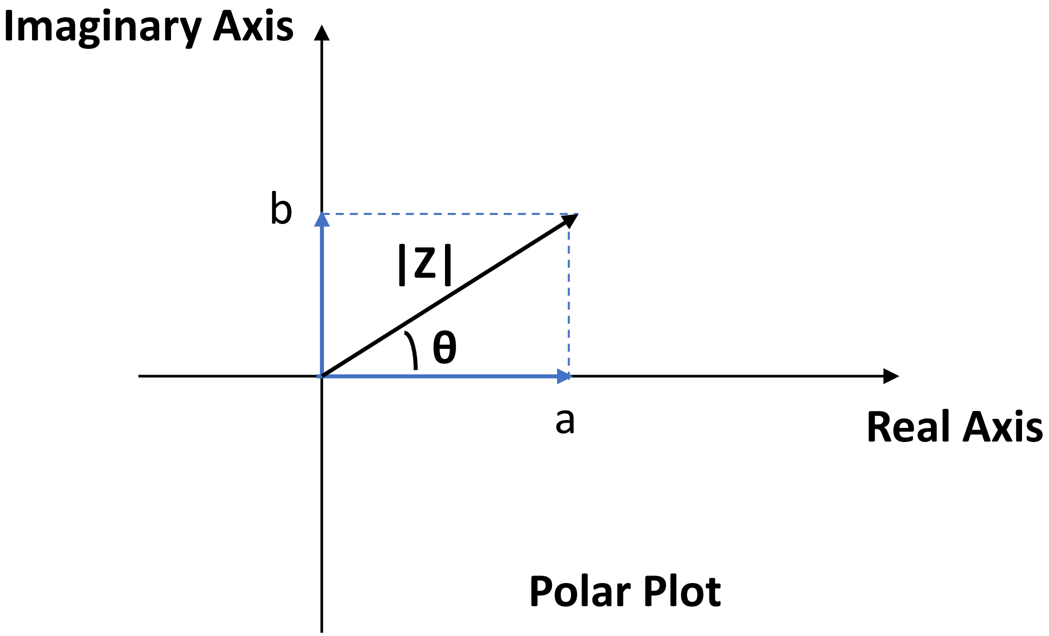
- So, when combining a capacitance, inductance and a resistance in any combination in a circuit, the analysis will involve basic complex number arithmetic.

Complex Numbers

- A complex number is one that is made of a real part and an imaginary. An imaginary part is one that could be a result of the square root of a negative number.
- A complex number can be represented in 2 formats:
  - Rectangular format as $Z = a + jb$ (where j indicates the imaginary number).
  - Polar Format as $Z = |Z|\angle\theta$ which is a magnitude at a certain angle.
- A complex number can be plotted on a complex number plane as shown:



Rectangular Plot



Polar Plot

Complex Number Conversions

- Given a complex number represented in the 2 formats:
  - Rectangular format as Z = a + jb (where j indicates the imaginary number).
  - Polar Format as $Z = |Z|\angle\theta$ which is a magnitude at a certain angle.

- To convert from rectangular to polar:
  - $|Z| = \sqrt{a^2 + b^2}$.
  - $\angle\theta = Tan^{-1}\left(\frac{b}{a}\right)$.

- To convert from polar to rectangular:
  - The real part is given by:
    - $a = |Z| \times \cos(\theta)$
  - The imaginary part is given by
    - $b = |Z| \times \sin(\theta)$

- Additions & Subtractions are faster/easier done in rectangular format.

- Divisions & Multiplications are faster/easier done in polar format.

Complex Number Addition, Subtraction, Multiplication & Division

- Given 2 complex numbers:
  - $Z_1 = a_1 + \mathrm{j}\, b_1 = |Z_1|\angle\theta_1$ & $Z_2 = a_2 + \mathrm{j}\, b_2 = |Z_2|\angle\theta_2$

  Then :

  For additions and subtractions:

  - $Z_1 + Z_2 = (a_1 + a_2) + \mathrm{j}(b_1 + b_2)$
  - $Z_1 - Z_2 = (a_1 - a_2) + \mathrm{j}(b_1 - b_2)$

  - For multiplications and divisions:
  - $Z_1 \times Z_2 = |Z_1|\angle\theta_1 \times |Z_2|\angle\theta_2 = |Z_1| \times |Z_2|\angle(\theta_1 + \theta_2)$ (Multiply the magnitudes and add the angles).

  - $\frac{|Z_1|\angle\theta_1}{|Z_2|\angle\theta_2} = \frac{|Z_1|}{|Z_2|}\angle(\theta_1 - \theta_2)$ (Divide the magnitudes and subtract the denominator angle from the numerator angle).

Complex Number Arithmetic Example

- Given 2 complex numbers: Z_1 = 2 + j 5, Z_2 = -3 – j8 & Z_3 = $|5|\angle 45^{\circ}$, Do the following:

a- Plot Z_1 , Z_2 & Z_3

b- Find $Z_1 + Z_2$

c- Find Z_1 - Z_2

d- Covert Both Z_1 & Z_2 to their polar format

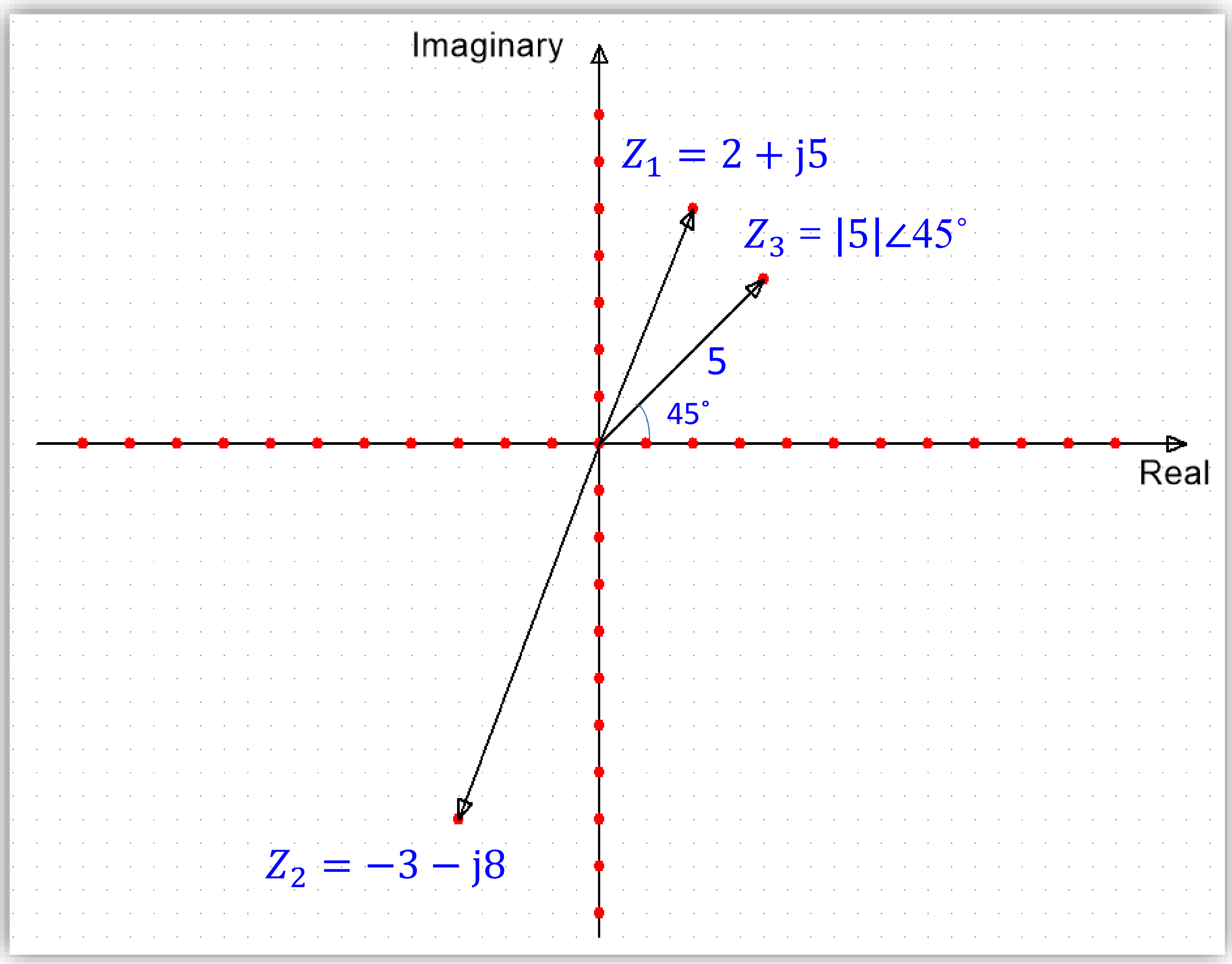
e- Convert Z_3 to its rectangular format

f- Find $Z_1 \times Z_2 \times Z_3$

g- Find $\frac{Z_1}{Z_2}$

Complex Number Arithmetic Example

a- Plot Z_1 , Z_2 & Z_3



Complex Number Arithmetic Example

b- $Z_1 + Z_2$

Z_1 = 2 + j 5

Z_2 = -3 – j8

- $Z_1 + Z_2$ = (2 - 3) + j(5 – 8)
- $Z_1 + Z_2$ = -1 - j3

Complex Number Arithmetic Example

c- Z_1 - Z_2

Z_1 = 2 + j 5

Z_2 = -3 – j 8

- Z_1 - Z_2 = (2 + 3) + j(5 + 8)

- Z_1 - Z_2 = 5 + j 13

Complex Number Arithmetic Example

d- Covert Both Z_1 & Z_2 to their polar format

- To convert from rectangular to polar:
  - $|Z| = \sqrt{a^2 + b^2}$.
  - $\angle\theta = Tan^{-1}\left(\frac{b}{a}\right)$.

- Z_1 = 2 + j 5
  - $|Z_1| = \sqrt{2^2 + 5^2} = \sqrt{29} = 5.38, \ \theta_1 = Tan^{-1}\left(\frac{b}{a}\right) = Tan^{-1}\left(\frac{5}{2}\right) = 63.43^\circ$
  - $Z_1 = |5.38|\angle 63.43^\circ$

- Z_2 = -3 – j 8
  - $|Z_2| = \sqrt{-3^2 + -8^2} = \sqrt{73} = 8.54, \ \theta_2 = Tan^{-1}\left(\frac{b}{a}\right) = Tan^{-1}\left(\frac{-8}{-3}\right) = 69.44^\circ$
  - But the complex number is in the third quadrant. Therefore a 180˚ should be subtracted from the answer to get (69.44 – 180)˚ = -110.55˚.

- therefore:
  - $Z_2 = |8.54| \angle{-110.55^\circ}$

Complex Number Arithmetic Example

e- Convert Z_3 to its rectangular format

- $Z_3 = |5|\angle 45^\circ$
- $|Z_3| = 5.$
- $\theta = 45^\circ$

- To convert from polar to rectangular:
  - The real part is given by:
    - $a = |Z_3| \times \cos(\theta) = 5 \times \cos(45^\circ) = 3.535.$
  - The imaginary part is given by
    - $b = |Z_3| \times \sin(\theta) = 5 \times \sin(45^\circ) = 3.535$
- Therefore:
  - $Z_3 = |5|\angle 45^\circ = 3.535 + j\,3.535$

Complex Number Arithmetic Example

f- Find $Z_1 \times Z_2 \times Z_3$

- Multiplication is done in polar format.
- $Z_1 \times Z_2 \times Z_3 = |Z_1|\angle\theta_1 \times |Z_2|\angle\theta_2 \times |Z_3|\angle\theta_3 = |Z_1| \times |Z_2| \times |Z_3| \angle(\theta_1 + \theta_2 + \theta_3)$ (Multiply the magnitudes and add the angles).
    - $Z_1 = |5.38|\angle 63.43^\circ$
    - $Z_2 = |8.54|\angle\text{-}110.55^\circ$
    - $Z_3 = |5|\angle 45^\circ$
- Therefore:
    - $Z_1 \times Z_2 \times Z_3 = |5.38|\angle 63.43^\circ \times |8.54|\angle\text{-}110.55^\circ \times |5|\angle 45^\circ$
    - $Z_1 \times Z_2 \times Z_3 = (5.38 \times 8.54 \times 5)\ \angle(63.43^\circ - 156.86^\circ + 45^\circ) = 229.73\ \angle\text{-}\ 2.12^\circ$

Complex Number Arithmetic Example

- Given 2 complex numbers: Z_1 = 2 + j 5, Z_2 = -3 – j8 & Z_3 = |5|∠45°, Find:

g- Find $\frac{Z_1}{Z_2}$

- Division is done in polar format.
- $\frac{|Z_1|\angle\theta_1}{|Z_2|\angle\theta_2} = \frac{|Z_1|}{|Z_2|}\angle(\theta_1 - \theta_2)$ (Divide the magnitudes and subtract the denominator angle from the numerator angle).

  - Z_1 = |5.38|∠63.43°

  - Z_2 = |8.54|∠-110.55°

- Therefore:
  - $\frac{Z_1}{Z_2} = \frac{|5.38|\angle 63.43^\circ}{|8.54|\angle -110.55^\circ} = \frac{5.38}{8.54}\angle(63.43^\circ - (-110.55^\circ)) = 0.62 \angle 173.98^\circ$

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