RC circuits under AC

Trigonometric Function arguments

- The argument of the sine or cosine waveforms or any trigonometric function is always an angle even though it may appear that it is a function of time. See the following:

  - $\omega t = 2\pi Ft$. ω is the angular frequency in radians and F is the frequency in Hz.

  - $F = \dfrac{1}{T\ in\ seconds}$

  Therefore:

  - $\omega t = 2\pi Ft = 2(\pi\ in\ radians) \times \dfrac{(t\ in\ seconds)}{(T\ in\ seconds)} = \dfrac{2\pi t}{T}\ Radians$

Complex Numbers Review

- Under AC conditions, the capacitor behaves as an ohmic value which is known as the capacitive reactance X_c.

- Such a reactance is manifested as an imaginary number.

- That coupled with the fact that resistances are considered real values dictates that a total ohmic value that encompasses resistances and capacitances will end up as a complex number.

- A complex number can be represented in 2 formats:
  - Rectangular format as Z = a + jb (where j indicates the imaginary number).
  - Polar Format as Z = $|Z|\angle\theta$ which is a magnitude at a certain angle.

Complex Number Conversions

- Given a complex number represented in the 2 formats:
  - Rectangular format as Z = a + jb (where j indicates the imaginary number).
  - Polar Format as Z = $|Z|\angle\theta$ which is a magnitude at a certain angle.

- To convert from rectangular to polar:
  - $|Z| = \sqrt{a^2 + b^2}$.
  - $\angle\theta = Tan^{-1}\left(\frac{|b|}{a}\right)$.

- To convert from polar to rectangular:
  - The real part is given by:
    - a = $|Z| \times \cos(\theta)$
  - The imaginary part is given by
    - b = $|Z| \times \sin(\theta)$

- Additions & Subtractions are easier done in rectangular format.

- Divisions & Multiplications are easier done in polar format.

Complex Number Addition, Subtraction, Multiplication & Division

- Given 2 complex numbers:
  - $Z_1 = a_1 + \text{j}\, b_1 = |Z_1|\angle\theta_1$ & $Z_2 = a_2 + \text{j}\, b_2 = |Z_2|\angle\theta_2$

  Then :

  For additions and subtractions:

  - $Z_1 + Z_2 = (a_1 + a_2) + \text{j}(b_1 + b_2)$
  - $Z_1 - Z_2 = (a_1 - a_2) + \text{j}(b_1 - b_2)$

  - For multiplications and divisions:
  - $Z_1 \times Z_2 = |Z_1|\angle\theta_1 \times |Z_2|\angle\theta_2 = |Z_1| \times |Z_2|\angle(\theta_1 + \theta_2)$ (Multiply the magnitudes and add the angles).

  - $\frac{|Z_1|\angle\theta_1}{|Z_2|\angle\theta_2} = \frac{|Z_1|}{|Z_2|}\angle(\theta_1 - \theta_2)$ (Divide the magnitudes and subtract the denominator angle from the numerator angle).

Voltage and Current Relationship in a Capacitor

- The capacitor will always try to oppose a change in the voltage across it by briefly holding its value.

- The capacitor will never oppose a current change through it.

- Because of the above two facts, the voltage across the capacitor will be held behind the current through the capacitor.

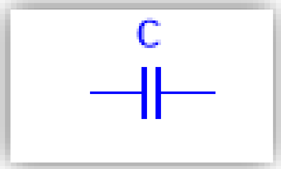
- In other words, we say that the capacitor voltage **lags** the current through it, or also stated as the current through the capacitor **leads** the voltage across it.

- **The capacitor voltage will always lag the current through it by 90 deg. Stated otherwise, the current through the capacitor will lead the voltage across it by 90 deg.**

The "ELI the ICE man" Memory Aid

- L is the inductor
- C is the capacitor
- E is the voltage
- I is the current
- Always the phase difference is 90˚.
- ELI means in an inductor L; the voltage E across it leads the current through it by 90˚.
- ICE means in a capacitor C, the current I through it leads the voltage E across it by 90˚.

The Capacitive Reactance (Impedance) Xc



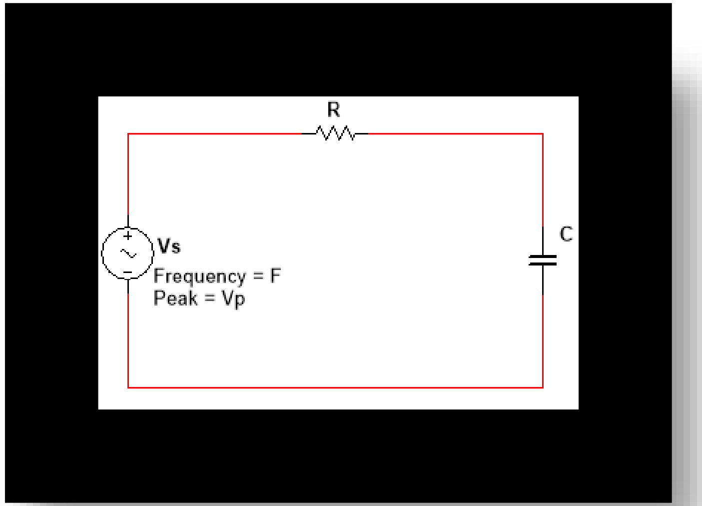
- The capacitive reactance of a capacitance C in an AC circuit with a frequency F is given by:
  - $X_C = \frac{1}{2\pi FC} = \frac{1}{\omega C}$ is the magnitude of the capacitive reactance.
  - Since ω = 2πF, ω is the angular frequency and F is the frequency.

- Since it is an imaginary value, it is expressed as:
  - -jX_C in rectangular format &
  - $|X_C|\angle$-90 Deg in Polar format.

- Expressed in time domain, this would be:
  - $X_C(t) = |X_C|$ sin (ωt - 90) = $\frac{1}{2\pi FC}$ sin (ωt - 90).

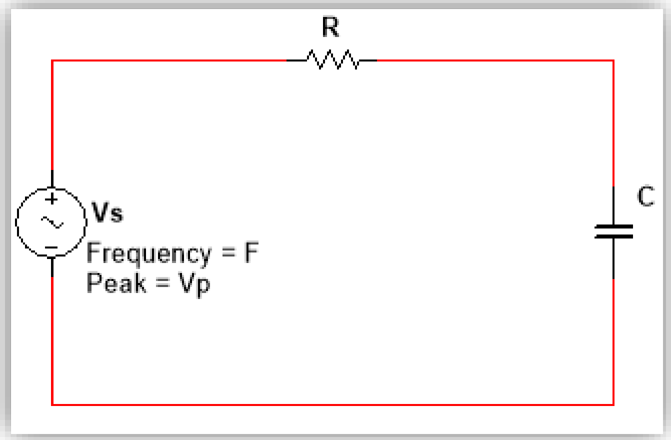
- **In a capacitor, always the voltage across the capacitor lags the current through it by 90°.**

Series RC circuits under AC



Series RC Circuits Under AC Equations

- For the series RC circuit shown:
- Vs (t) = Vp × sin (2πFt) = Vp × sin (ωt) = Vp ∠*0 deg*



- The magnitude value of the capacitive reactance is given by: $X_C = \frac{1}{2\pi FC}$.

- Since this is a series circuit, the total impedance seen by the voltage source Z_T is the sum of both R & Xc (not algebraically) and it is given by:

- $Z_T = R - jX_C$, represented in rectangular format.

- $Z_T = |Z_T|\angle\theta$ represented in polar format where:
  - $|Z_T| = \sqrt{R^2 + Xc^2}$ for the magnitude of the polar form.
  - $\theta = \tan^{-1}(\frac{-Xc}{R})$ for the phase angle for the polar form.

- This angle indicates that the total current I_T is leading the total voltage V_S by a value of $|\theta|$. This is always the case in a capacitive circuit.

Series RC Circuits Under AC Equations Continued

- The total current I_T For the series RC is given by:
  - $I_T = \frac{V_T}{Z_T} = \frac{V_S}{Z_T}$
  - $I_T = I_R = I_C$

- The voltage across the resistance is given by:
  - $V_R = I_R \times R = I_T \times R$
  - The voltage across a resistance and the current through it are in phase.

- The voltage across the capacitor is given by:
  - $V_C = I_C \times X_C = I_T \times X_C$
  - The voltage across the capacitor and the current through it are $90^\circ (\frac{\pi}{2})$ out of phase; with the current through the capacitor leading the voltage across it.

- The total applied voltage to the R-C series combination in terms of the voltages across R and across C is given by:
  - $V_T = \sqrt{{V_R}^2 + {V_C}^2}$

Phasor Diagrams

- For this series RC circuits under AC conditions, all currents and voltages are merely AC waveforms.

- To plot those sine waves vs time, it might be a bit cluttered to see the phase relationship between all those waveforms.

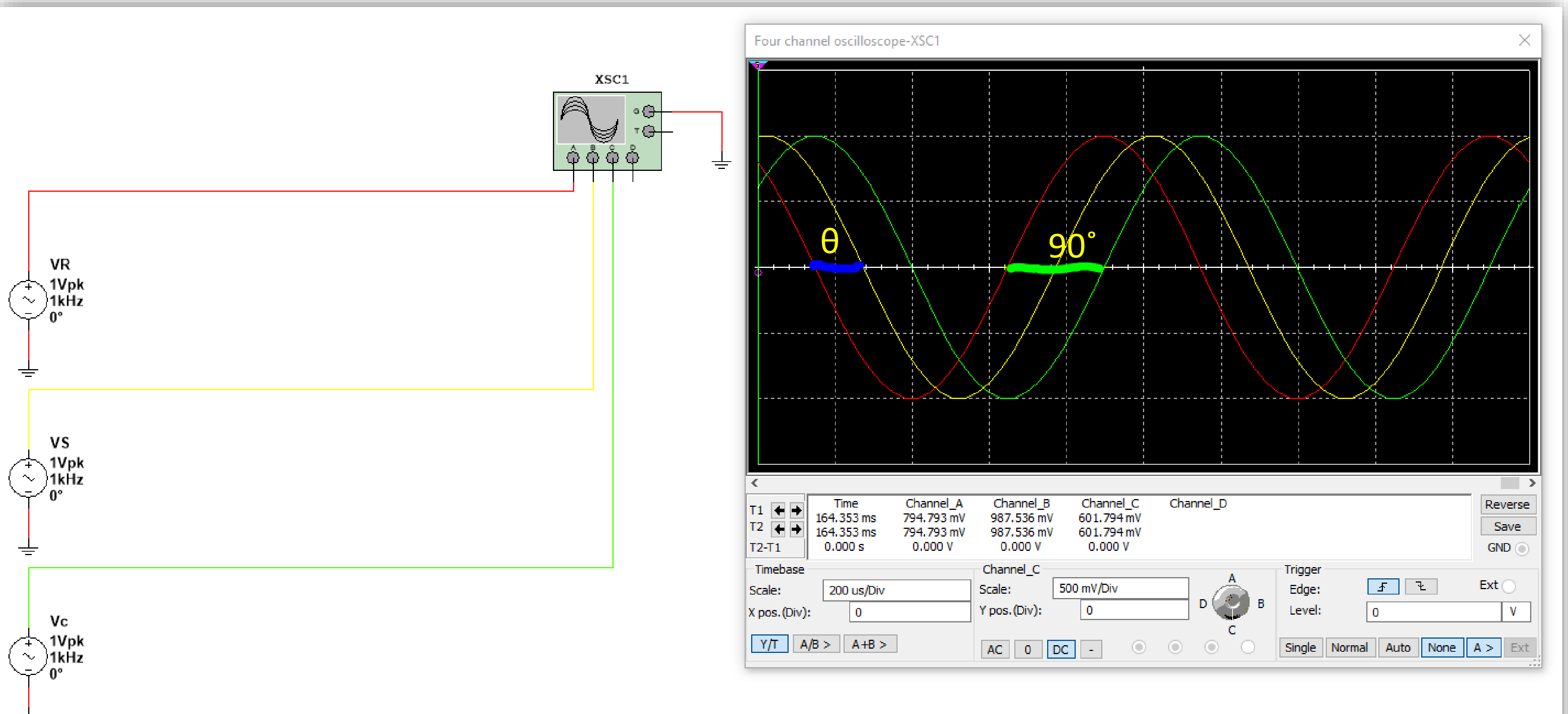
- It may be even much harder for an AC circuit to represent on a plot especially if the circuit becomes more complex and not merely a series RC circuit.

- Recall that the series current leads the source voltage by a value of θ and the current through the cap which is in phase with the voltage across the resistance leads the voltage across the cap by 90°.

- Phasor diagram solve this issue by simply representing the sine waves as arrows that represent the peak value and the phase angle.

Phasor Diagrams

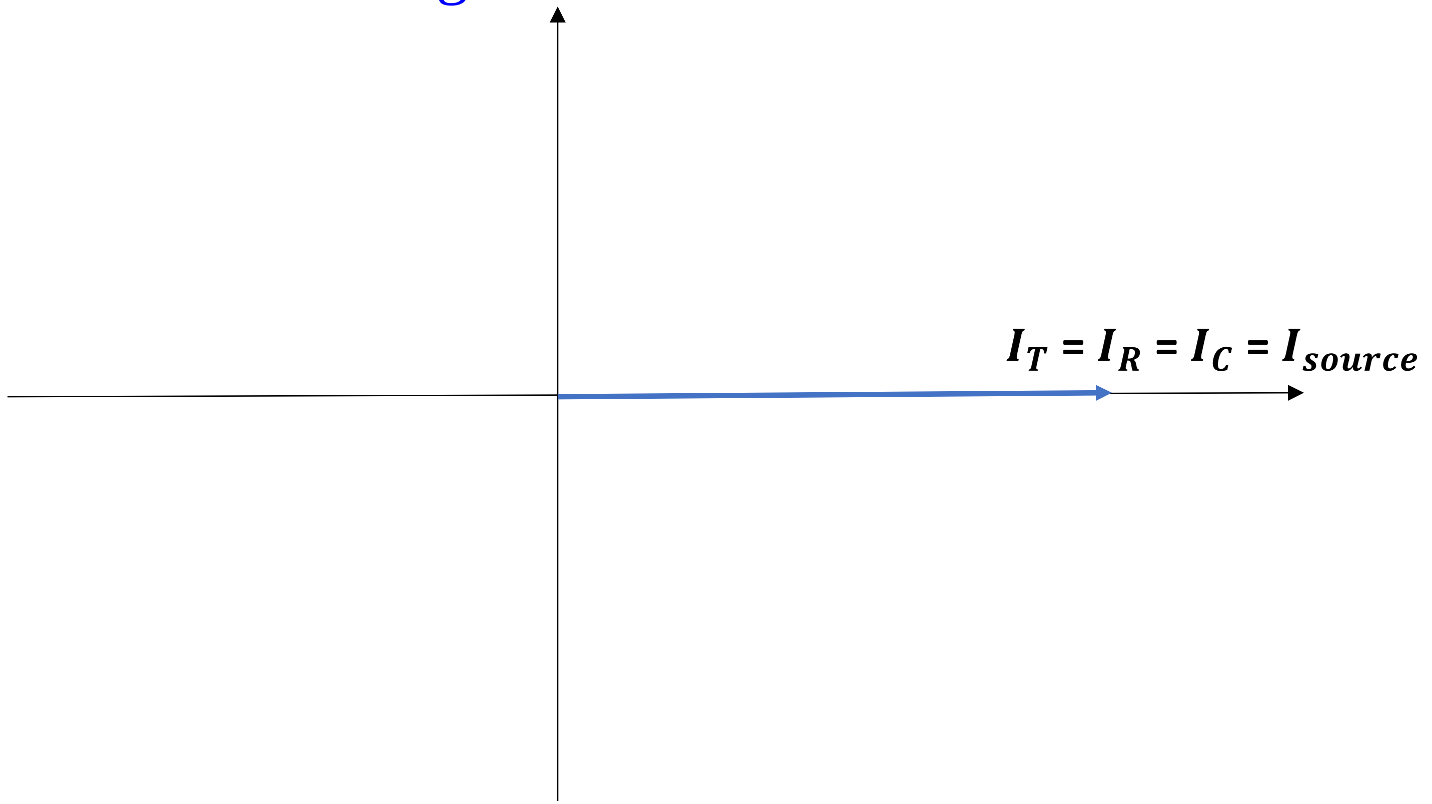
- The following figure shows a plot of the 3 sine waves, V_S (Yellow) , V_R (Red) and V_C (Green).



In a series RC circuit, the current is common to all element. Therefore, I(resitor) = I(source) = I(capacitor).
Notice that the current through the series RC circuit leads the voltage source Vs by a value of θ and the voltage across the cap by 90 degrees.

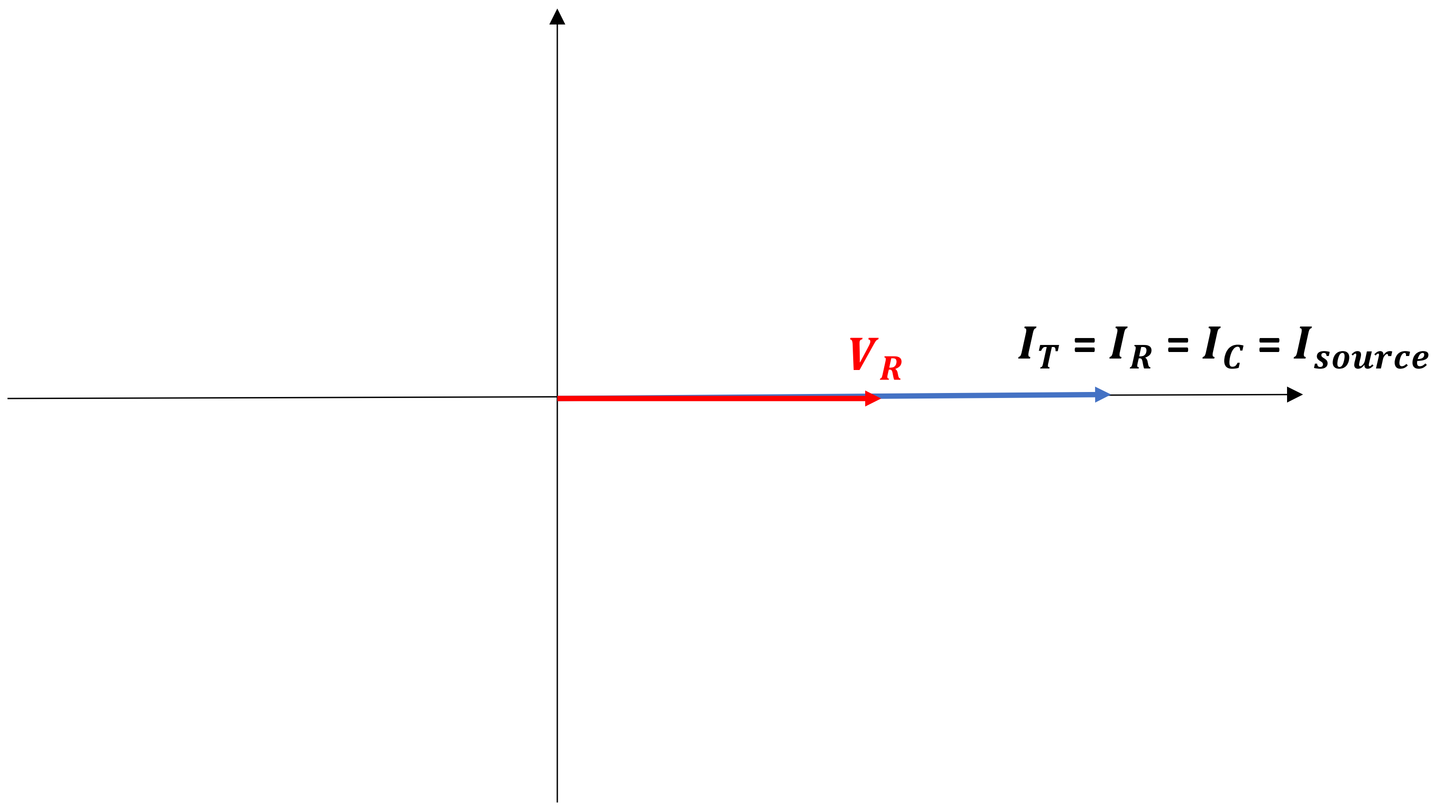
Phasor Diagrams

- For this series RC circuits under AC conditions, to draw the phasor diagram look for a signal that is common to all elements. That would the current through the circuit. This current would be used as the reference signal.



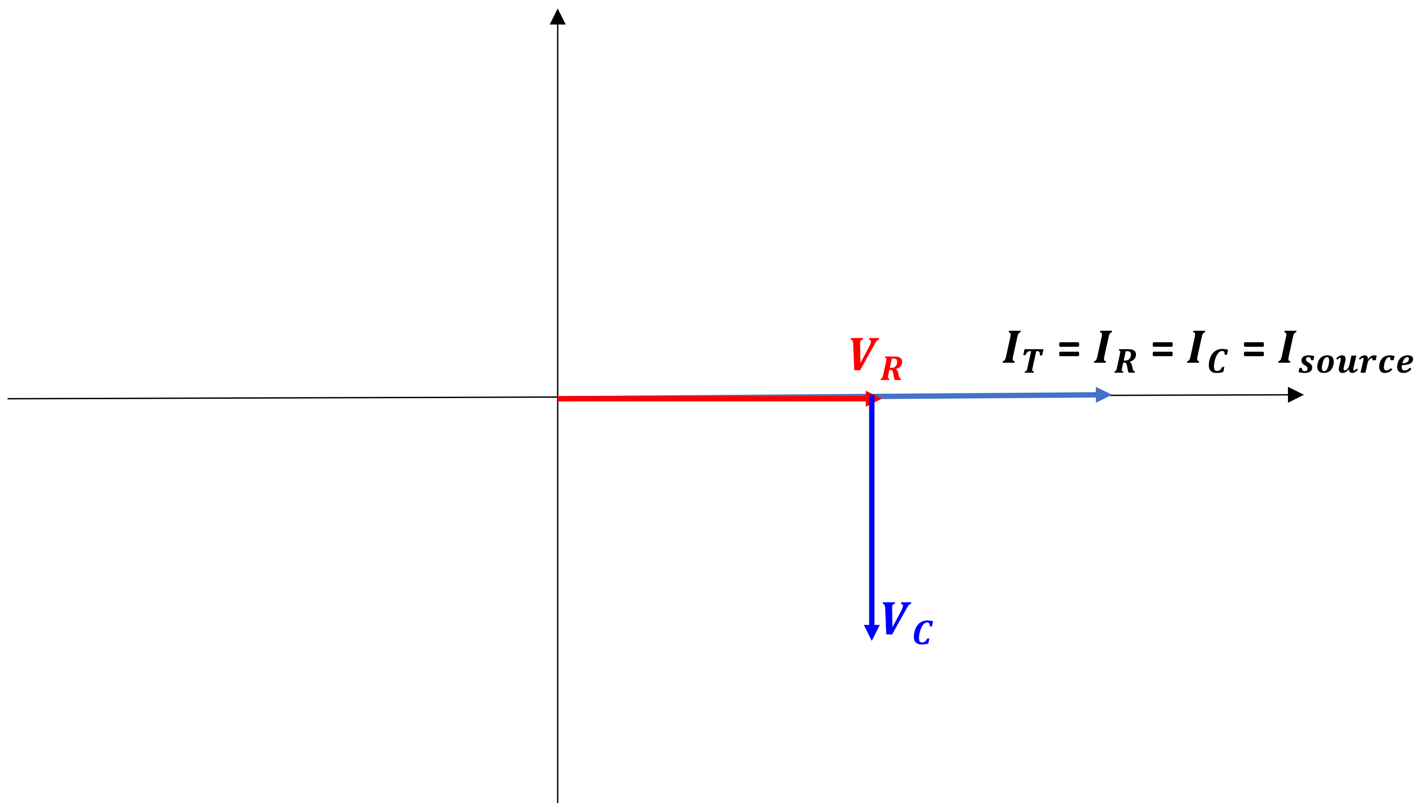
Phasor Diagrams Continued

- Since the voltage across R is in phase with the current through R, then they should line up as shown.



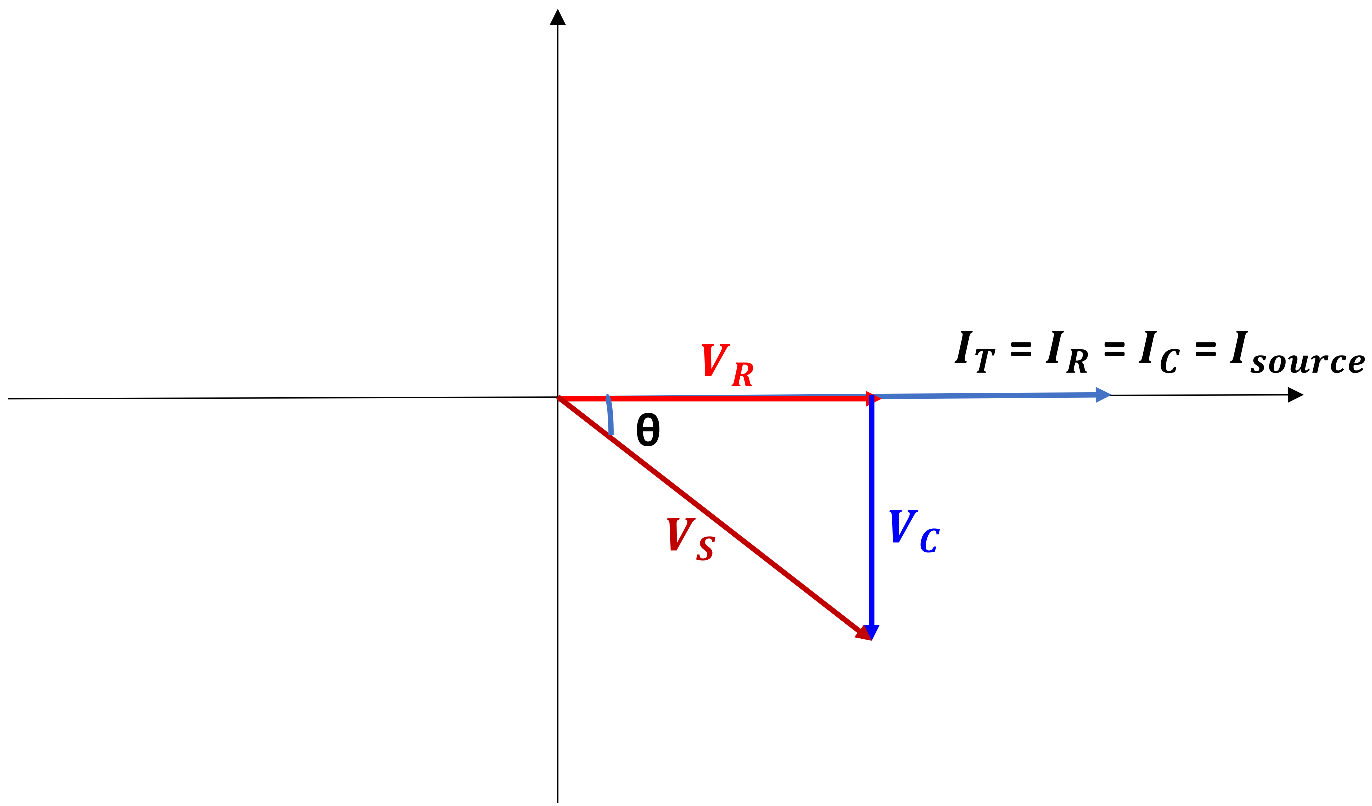
Phasor Diagrams Continued

- Since the voltage across the capacitor lags the current through it by 90˚, then it would be drawn as orthogonal to the current in a manner that is lagging by 90˚as shown.



Phasor Diagrams Continued

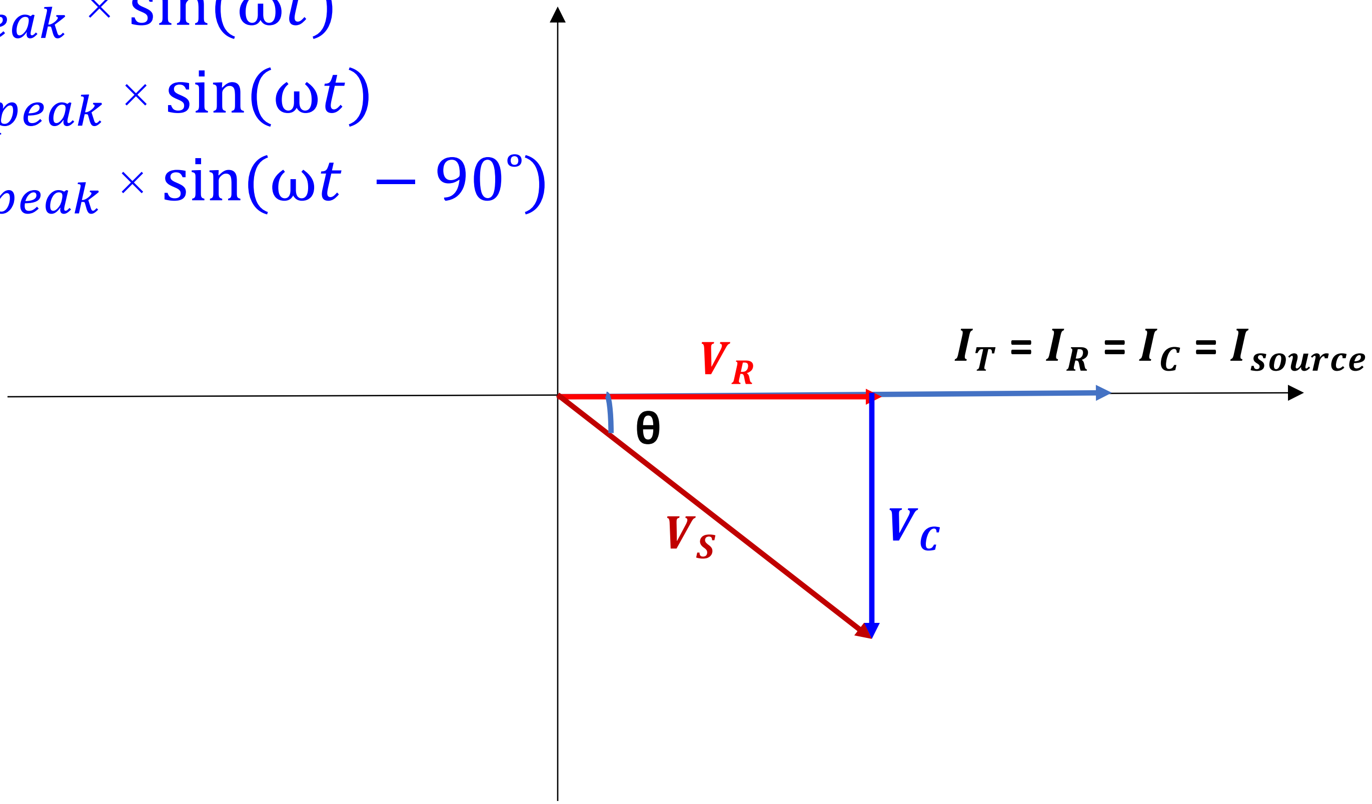
- Based on KVL, the source voltage is the sum of the voltage across R and the voltage across C. It would be represented as their vector sum and is drawn as shown on the phasor diagram.



- Notice that the total current leads the total voltage by an angle of θ which is the case in a series RC circuit.

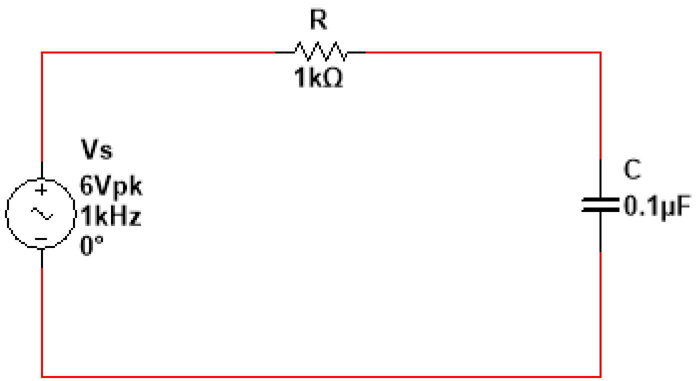
Phasor Diagrams Continued

- It is to note that each arrow represent a sine wave in the time domain:
  - $V_S(\mathrm{t}) = V_{S-peak} \times \sin(\omega t\ -\ \theta)$
  - $I_S\,(\mathrm{t}) = I_{S-peak} \times \sin(\omega t)$
  - $V_R\,(\mathrm{t}) = V_{R-peak} \times \sin(\omega t)$
  - $V_C\,(\mathrm{t}) = V_{C-peak} \times \sin(\omega t\ - 90^{\circ})$



Series RC Circuits Under AC Example

- For the following circuit find:
  - *a)* X_C both in rectangular and polar forms.
  - *b)* Z_T seen by the voltage source, both in polar & rectangular forms.
  - *c)* I_T, V_R , V_C and the phase difference between V_S & I_T .
  - *d)* Verify that the total voltage V_S is the sum of V_R & V_C (not the algebraic sum).
  - *e)* Draw the phasor diagram showing the total current and all voltages.



F = 1Khz, Vp = 6V

Series RC Circuits Under AC Example Solution

- Solution:

a) X_C both in rectangular and polar forms.

- X_C = 1/2πFC = 1/(2π × 1 KHz × 0.1 uF) = 1591Ω = 1.59 KΩ. Magnitude value.
  - In rectangular format it is –j 1.59 KΩ.
  - In Polar Format it is 1.59 KΩ $\angle$*-90˚*

b) Z_T both in polar & rectangular forms. Since the resistance and the capacitance are in series, the total impedance is given by their sum **(not algebraic sum, because R is real and Xc is imaginary)**

In rectangular format:

- Z_T = R – jXc = 1K - j1.59K

In polar format:

- $|Z_T| = \sqrt{R^2 + Xc^2} = \sqrt{1K^2 + (-1.59K)^2} = \sqrt{3.528}$ = 1.88 KΩ Magnitude.
- $\theta = \tan^{-1}(\frac{-|Xc|}{R}) = \theta = \tan^{-1}(\frac{-1.59}{1}) = -57.83\ ^o$. Which is Always the Phase Angle between the voltage source and the total current I_T.

Series RC Circuits Under AC Example Solution Continued

- Solution:

  c) I_T , V_R , V_C and the phase difference between V_S & I_T.

  - Since divisions & multiplications are easier performed in polar format:
  - $I_T = \dfrac{\text{Peak of } v_S\angle 0}{z_T \text{ in polar Format}} = \dfrac{v_P}{|z_T|} = \dfrac{v_P v}{|z_T|\angle\theta} = \dfrac{6\angle 0}{1.88\angle -57.83^\circ} = 3.19 \text{ mA} \angle 57.83^\circ$
  - In time domain $I_T(t) = 3.19\ mA \sin(2\pi Ft + 57.83^\circ)$
  - Since this is a series circuit $I_T = I_R = I_C = 3.19 \text{ mA} \angle 57.83^\circ$ Therefore:
  - $V_R = I_R \times R = 3.19 \text{ mA} \angle 57.83^\circ \times 1\ K\Omega = 3.19\ V \angle 57.83^\circ$
  - In time domain $V_R(t) = 3.19\ V \sin(2\pi Ft + 57.83^\circ)$
  - $V_C = I_C \times X_C = 3.19 \text{ mA} \angle 57.83^\circ \times 1.59\ K\Omega\angle -90^\circ = 5.07\ V\angle(57.83^\circ - 90^\circ) = 5.07\ V\angle -32.17^\circ$
  - In time domain $V_C(t) = 5.07\ V \sin(2\pi Ft - 32.17^\circ)$

- Notice that the difference between the current through C and the voltage across it is [57.83° – (-32.17°) = 90°] with the current leading the voltage.

- Since V_S is at 0° phase angle and the current is at 57.83°, then the current leads the voltage source by 57.83°. This enforces the fact that in a capacitive circuit, the total current leads the voltage source by θ.

Series RC Circuits Under AC Example Solution Continued

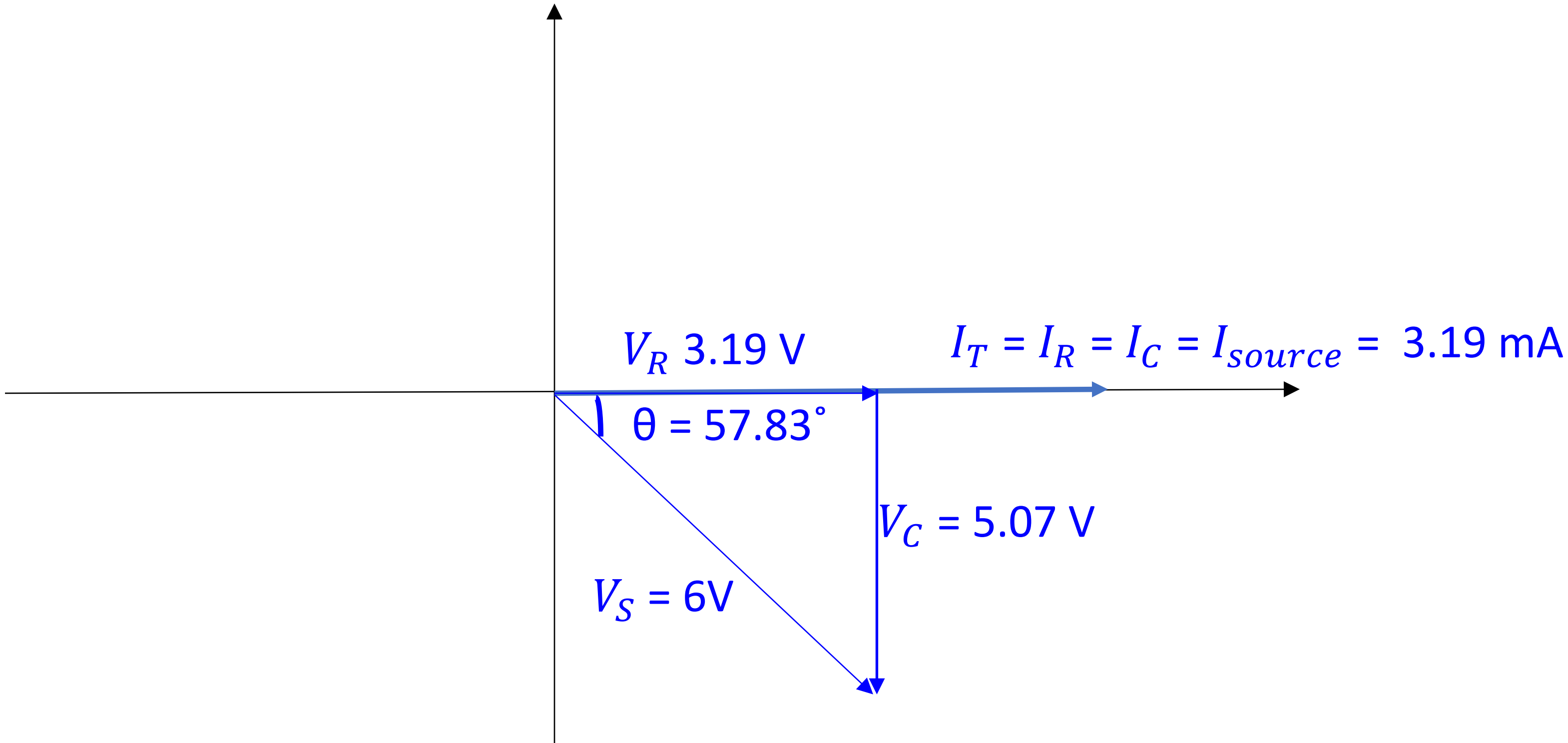
- Solution:

  d) Verify that the total voltage V_S is the sum of V_R & V_C.

- The total applied voltage to the R-C series combination in terms of the voltages across R and across C is given by:
  - $V_T = \sqrt{{V_R}^2 + {V_C}^2}$
  - $V_T = \sqrt{(3.19)^2 + (5.07)^2} \approx 6\,V.$

Series RC Circuits Under AC Example Solution Continued

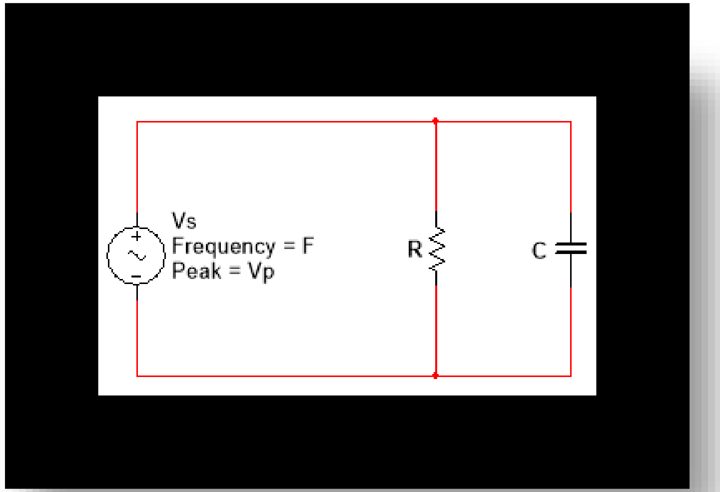
e) Draw the phasor diagram showing the total current and all voltages.



- Notice that the total current leads the total voltage by an angle of θ = 57.83˚ which is the case in a series RC circuit.

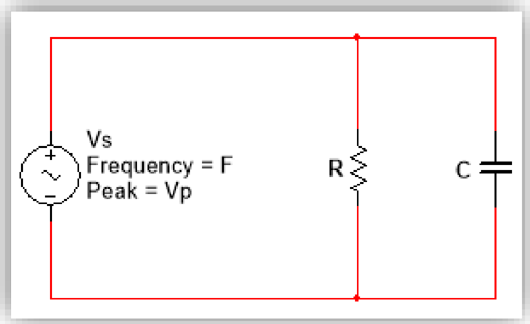
Parallel RC circuits under AC

Parallel RC circuits under AC



Parallel RC Circuits Under AC Equations

- For the series RC circuit shown:
- Vs = Vp × sin (2πFt) = Vp $\angle 0\ deg$



- The magnitude value of the capacitive reactance is given by: $X_C = \frac{1}{2\pi FC}$.
- Since this is a parallel circuit, the total impedance seen by the voltage source Z is the product of both R & Xc divided by their sum (not algebraically) and it is given by:
- $Z_T = \frac{R \times (-jX_C)}{R - jX_C}$, represented in rectangular format.
- $Z_T = |Z_T|\angle\theta$ represented in polar format where:
    - $|Z_T| = \frac{R \times X_C}{\sqrt{R^2 + Xc^2}}$ for the magnitude for the polar form.
    - $\theta = \tan^{-1}(\frac{R}{Xc})$ for the phase angle for the polar form.
- This angle indicates that the total current I_T is leading the total voltage V_S by a value of $|\theta|$. This is always the case in a capacitive circuit.

Series RC Circuits Under AC Equations Continued

- The total current I_T For the parallel RC is given by:
  - $I_T = \frac{V_T}{Z_T} = \frac{V_S}{Z_T}$
  - $V_s = V_R = V_C$

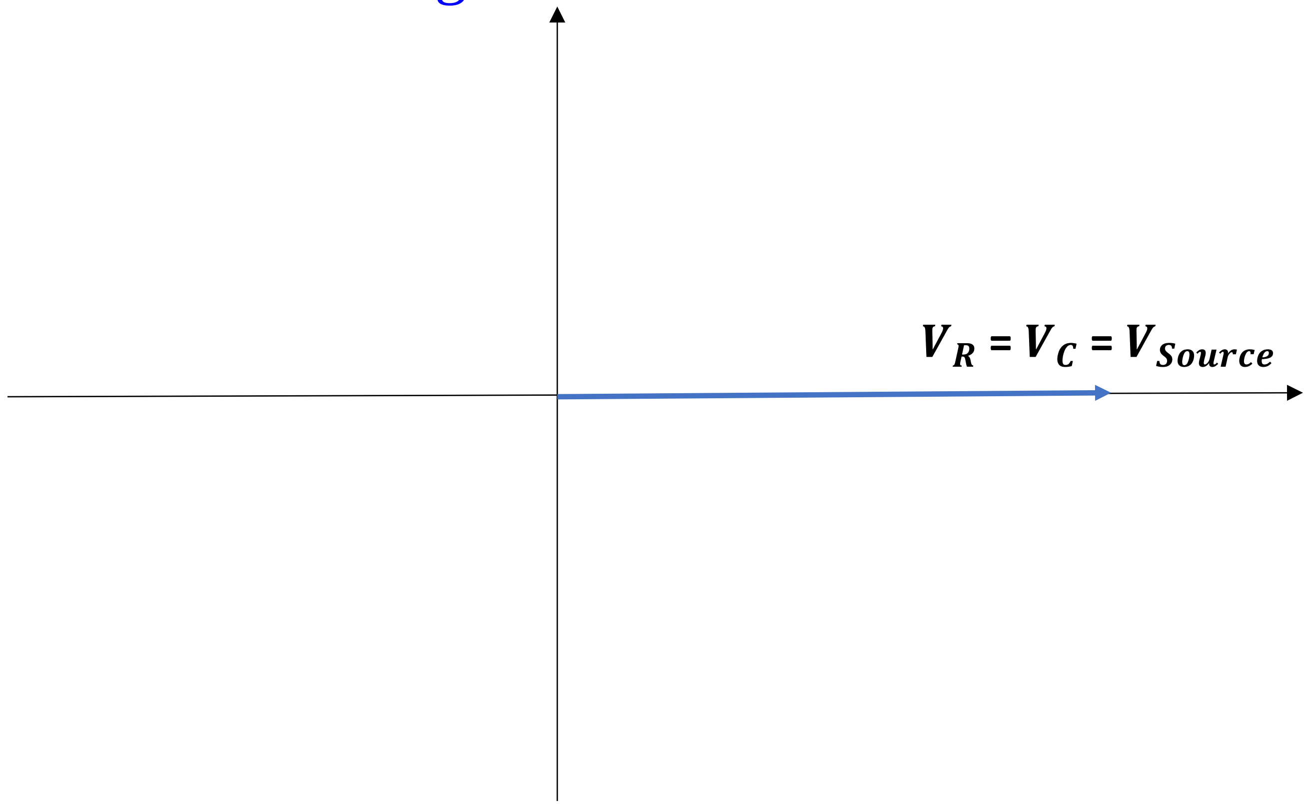
- The current through the resistance is given by:
  - $I_R = \frac{V_R}{R}$
  - The voltage across a resistance and the current through it are in phase.

- The current through the capacitor is given by:
  - $I_C = \frac{V_C}{X_C}$
  - The voltage across the capacitor and the current through it are 90˚$(\frac{\pi}{2})$ out of phase with the current through the capacitor leading the voltage across it.

- The total current I_T sourced by the voltage source in terms of the resistance and the capacitance current is given by:
  - $I_T = \sqrt{{I_R}^2 + {I_C}^2}$

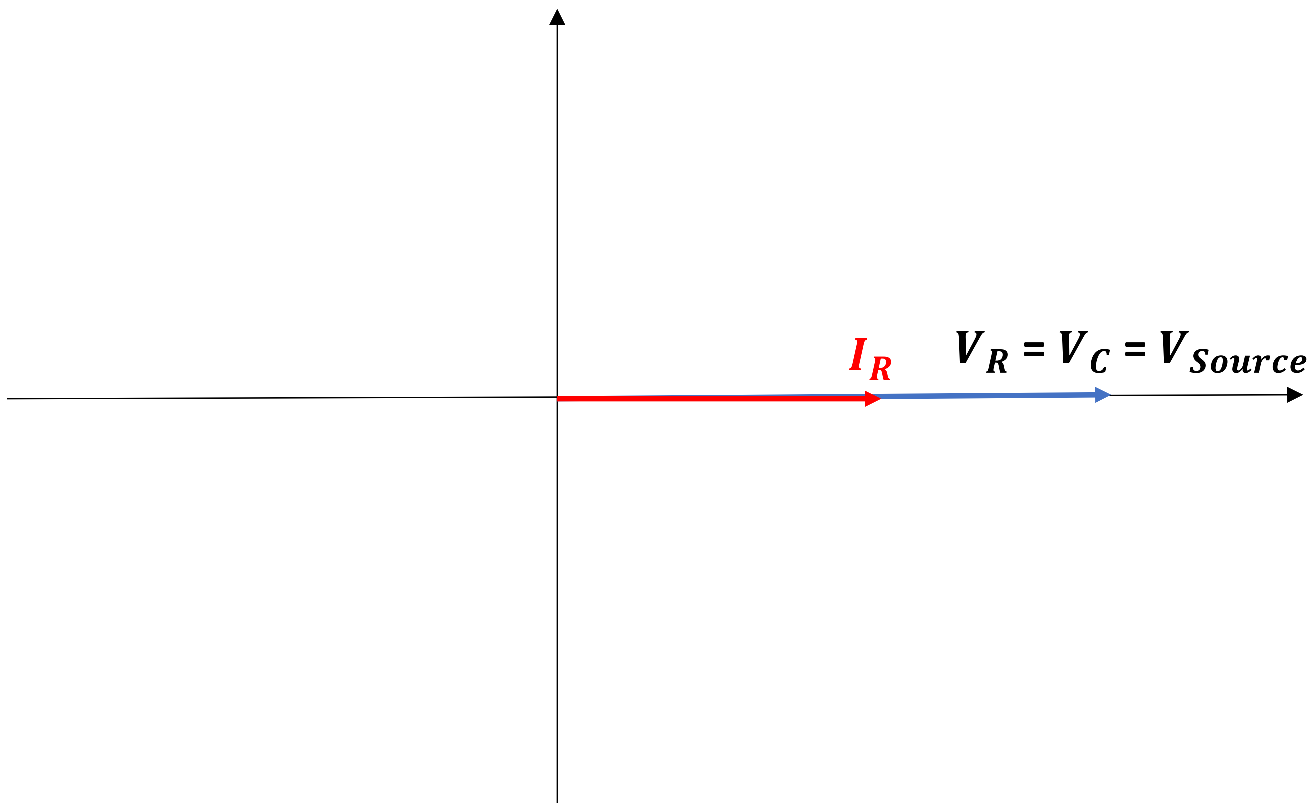
Phasor Diagrams

- For this parallel RC circuits under AC conditions, to draw the phasor diagram look for a signal that is common to all elements. That would the voltage across all elements V_s , V_R and V_C , in the circuit. This voltage would be used as the reference signal.



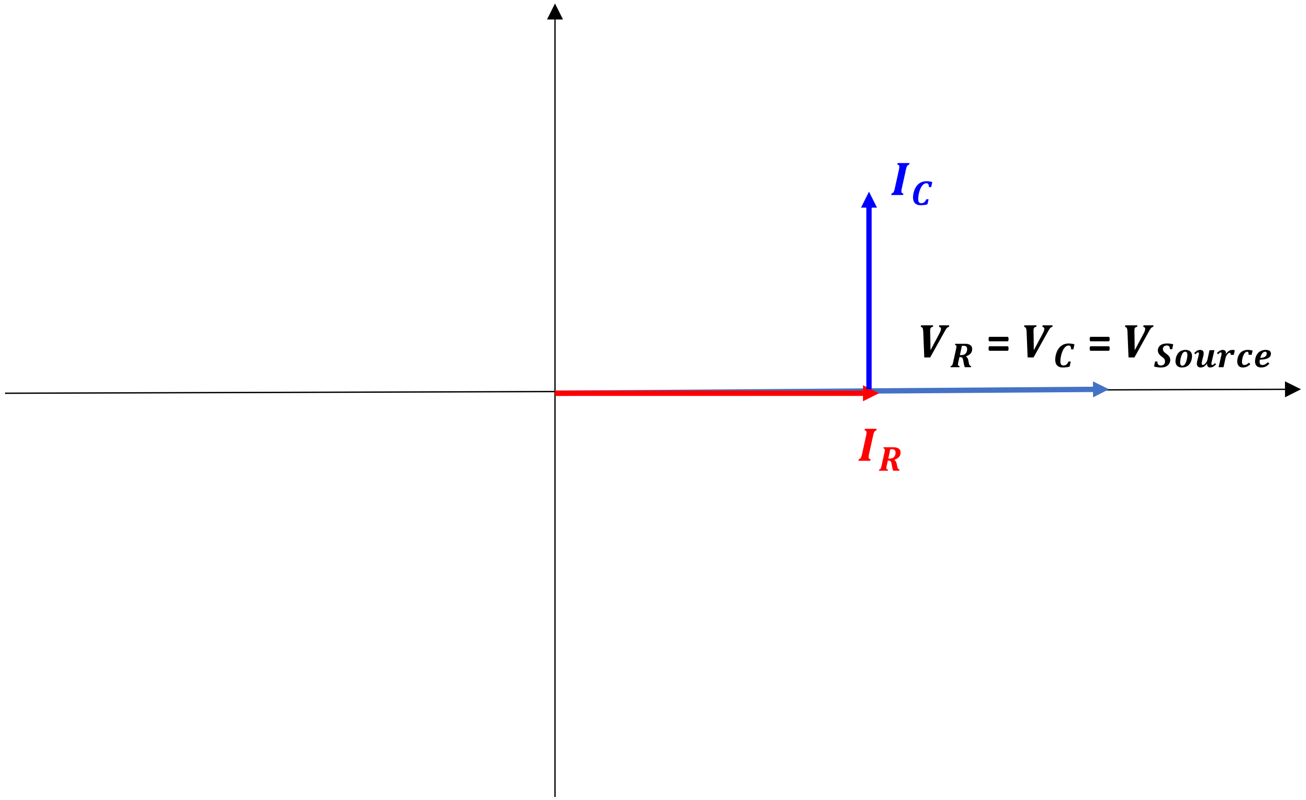
Phasor Diagrams Continued

- Since the current through R is in phase with the voltage across it, then they should line up as shown.



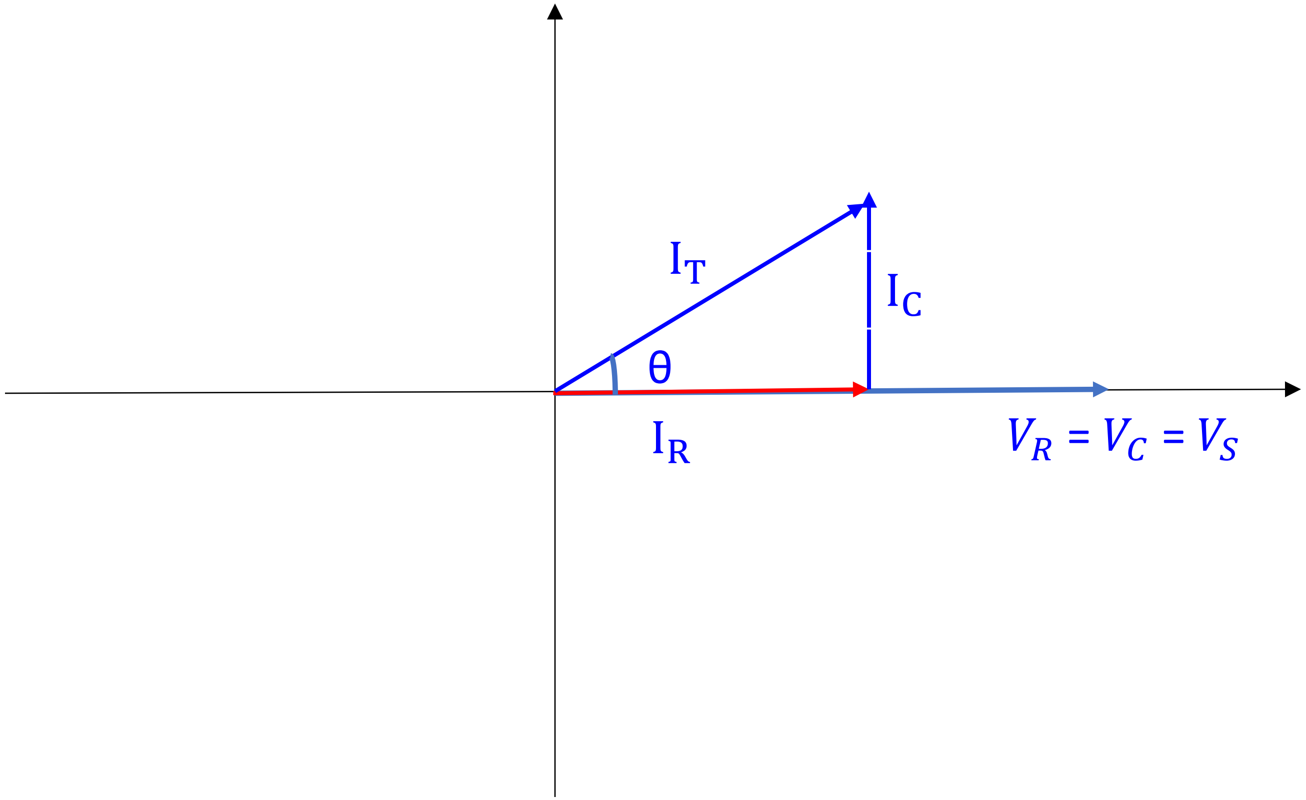
Phasor Diagrams Continued

- Since the current through the capacitor leads the voltage across it by 90˚, then it would be drawn as orthogonal to the voltage across it in a manner that is leading by 90˚ as shown.



Phasor Diagrams Continued

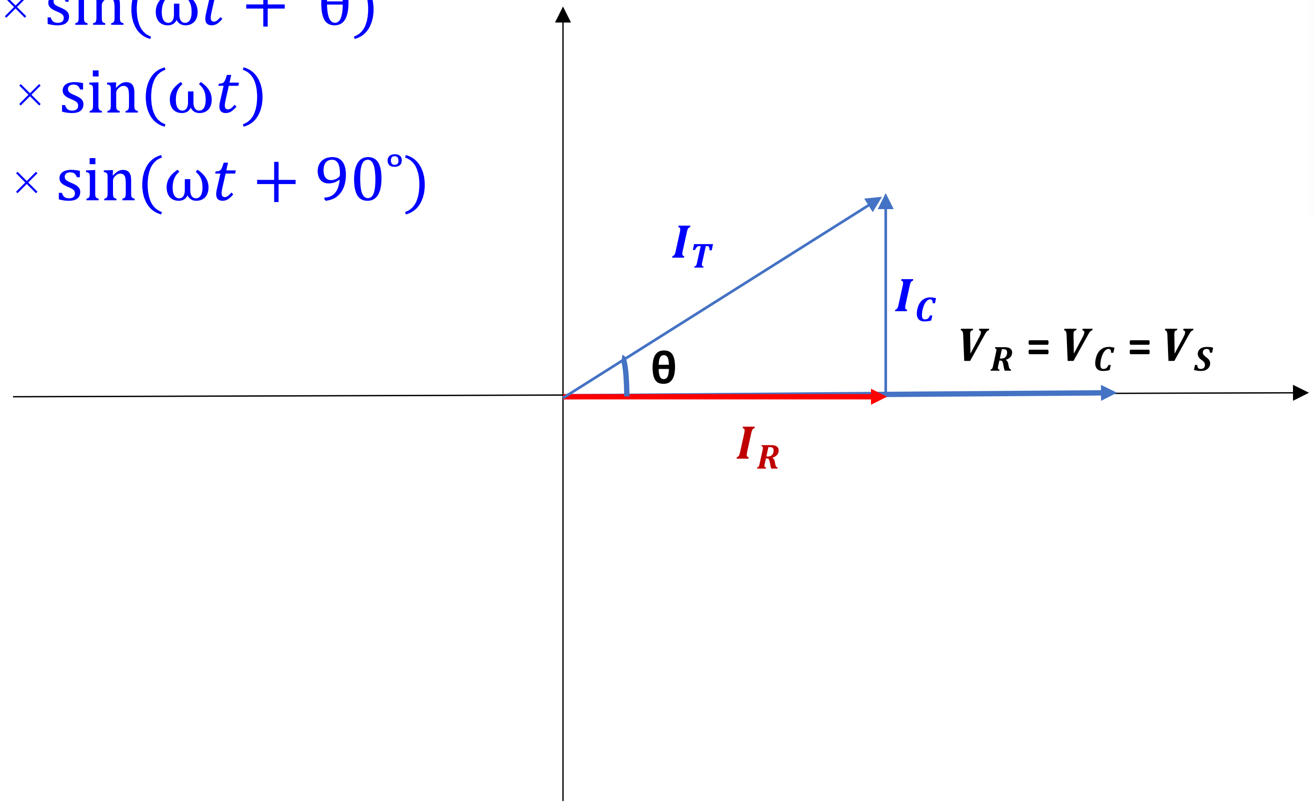
- Based on KCL, the source current is the sum of the currents through R and C. It would be represented as their vector sum and is drawn as shown on the phasor diagram.

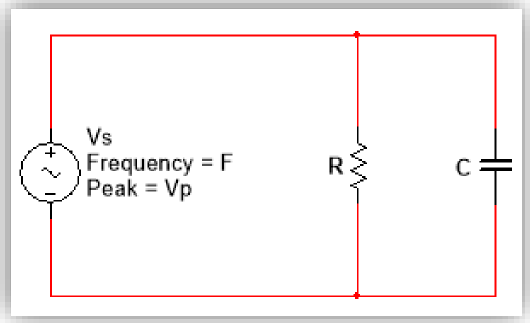


- Notice that the total current leads the total voltage by an angle of θ which is the case in a series RC circuit.

Phasor Diagrams Continued

- It is to note that each arrow represent a sine wave in the time domain:
  - $V_S(\mathrm{t}) = V_R(\mathrm{t}) = V_C(\mathrm{t}) = V_{S-peak} \times \sin(\omega t)$
  - $I_S = I_{S-peak} \times \sin(\omega t + \theta)$
  - $I_R = I_{R-peak} \times \sin(\omega t)$
  - $I_C = I_{C-peak} \times \sin(\omega t + 90°)$

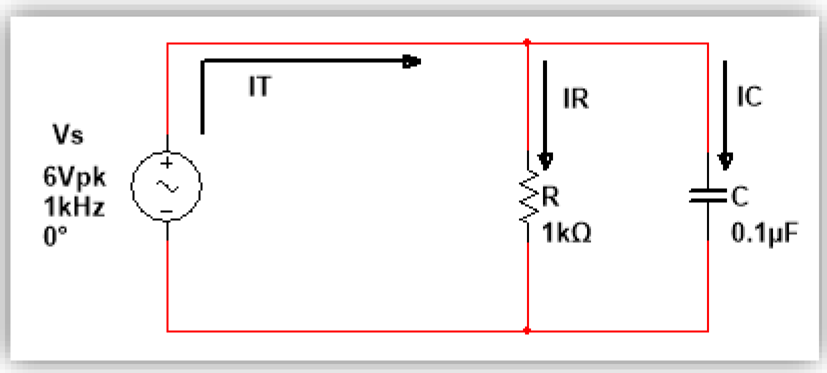




- Notice that the peaks of the voltages for all 3 elements are equal:
  - $V_{S-peak} = V_{R-peak} = V_{C-peak}$

Parallel RC Circuits Under AC Example

- For the following circuit find:
  - *a)* X_C both in rectangular and polar forms.
  - *b)* Z_T seen by the voltage source, both in polar & rectangular forms.
  - *c)* I_T, I_R, I_C and the phase difference between V_S & I_T.
  - *d)* Verify that the total current I_T(I_S) is the sum of I_R & I_C (not the algebraic sum).
  - *e)* Draw the phasor diagram showing the total voltage and all currents.



Parallel RC Circuits Under AC Example Solution

- Solution:

a) X_C both in rectangular and polar forms.

- $X_C = \frac{1}{2\pi FC} = \frac{1}{2\pi \times 1\ KHz \times 0.1\ \mu F} = 1.59\ K\Omega.$
  - In rectangular format it is –j 1.59 KΩ.
  - In Polar Format it is 1.59 KΩ $\angle$-90˚

b) Z_T both in polar & rectangular forms. Since the resistance and the capacitance are in parallel, the total impedance is given:

- In rectangular format:

$$Z_T = \frac{R \times (-jX_C)}{R - jX_C} = \frac{1 \times (-j1.59)}{1 - j1.59} = \frac{-j1.59}{1 - j1.59} = \frac{-j1.59\,(1 + j1.59)}{(1 - j1.59)(1 + j1.59)} = \frac{2.52 - j1.59}{3.53} = 0.712 - j0.45$$

- In polar format, simply convert the rectangular format to its polar format to obtain:
  - $|\ Z_T\ | = \sqrt{0.713^2 + 0.45^2} = 0.846$
  - $\theta = tan^{-1}\left(\frac{-045}{0.712}\right) = -32.16^{\circ}$

- We could have also found the polar format values using:
  - $|Z_T| = \frac{R \times X_C}{\sqrt{R^2 + Xc^2}} = \frac{1 \times 1.59}{\sqrt{1^2 + 1.59^2}} = 0.846$
  - $\theta = \tan^{-1}(\frac{-R}{X_C}) = \tan^{-1}(\frac{-1}{1.59}) = -32.16\ ^{o}$.

  - Which is Always the Phase Angle between the voltage source and the total current I_T.

Parallel RC Circuits Under AC Example Solution

c) Find I_T, I_R , I_C and the phase difference between V_S & I_T.

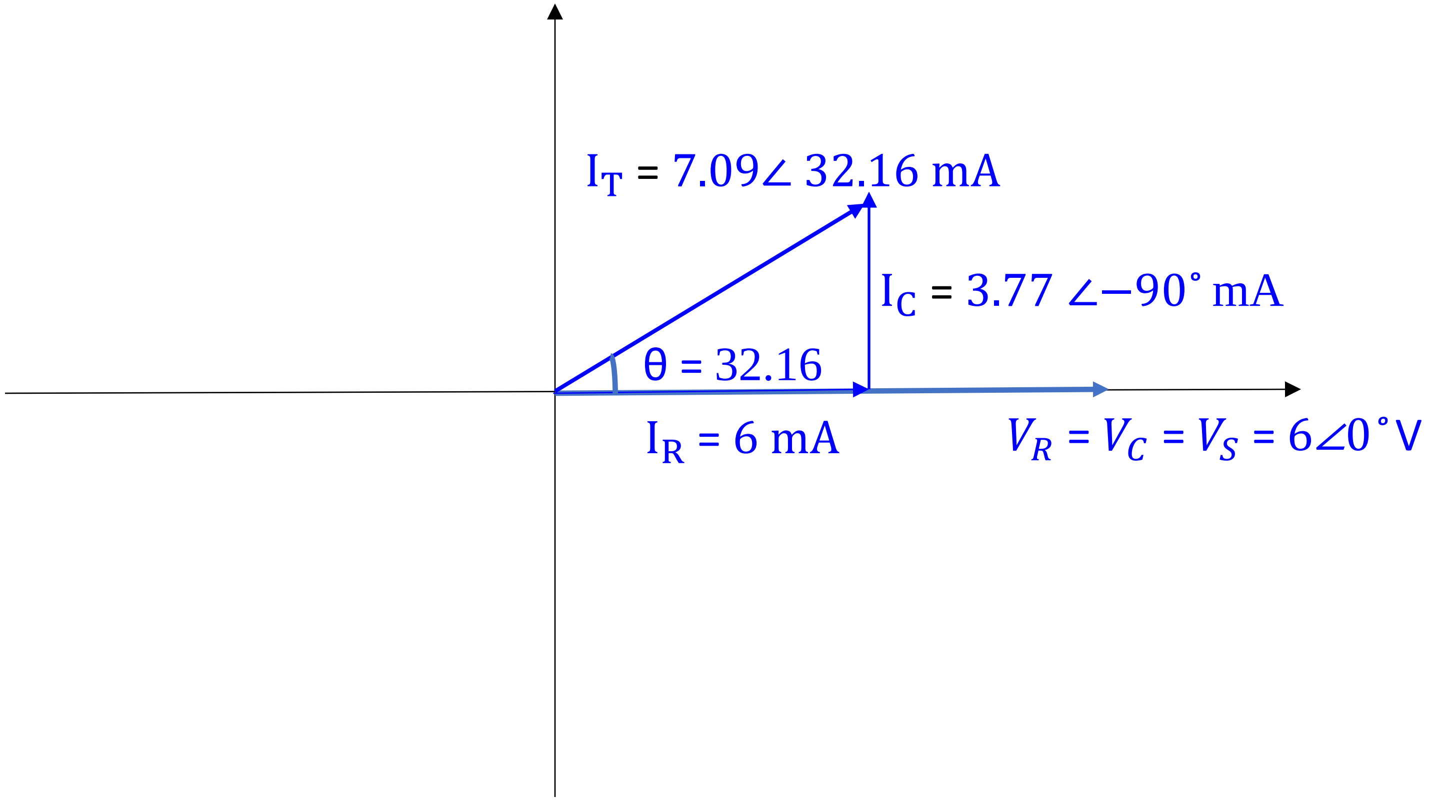
- Since this is a parallel circuit, then: $V_S = V_R = V_C = 6\angle 0^\circ V$.
- The total current I_T is equal to the total voltage divided by the total impedance Z_T.
  - $I_T = \frac{V_S}{Z_T} = \frac{6\angle 0^\circ}{0.846\ K\Omega\angle -32.16^\circ} = 7.09\angle\ 0 -(-32.16) = 7.09\angle\ 32.16\ mA$
- The current through the resistance is equal to $\frac{V_R}{R} = \frac{6\angle 0^\circ}{1K\Omega} = 6\ mA$
- The current through the capacitor is equal to $\frac{V_C}{X_C} = \frac{6\angle 0^\circ}{1.59\ K\Omega\angle -90^\circ} = 3.77\ \angle -90^\circ\ mA$
- The phase difference between V_T and I_T is equal to θ = 32.16 with the total current leading the total voltage.

d) Verify that the total voltage I_S is the sum of I_R & I_C (not the algebraic sum).

- To verify that the total current I_S is equal to the sum of I_R & I_C , use:
  - $I_T = \sqrt{{I_R}^2 + {I_C}^2} = \sqrt{6\ mA^2 + 3.77mA^2} = 7.09$ which is what I_T is equal to.

Phasor Diagrams Continued

e) Draw the phasor diagram showing the total voltage and all currents.

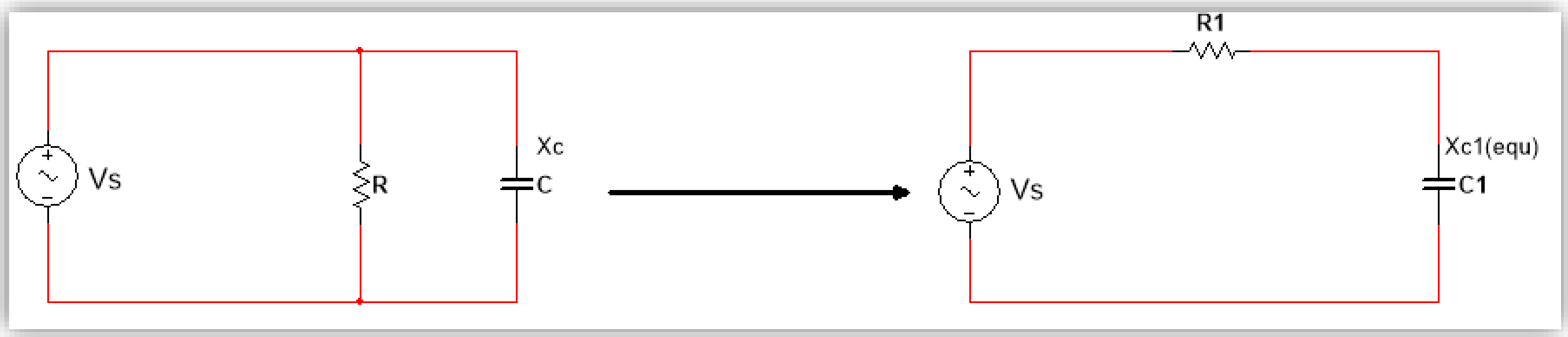


- Notice that the total current leads the total voltage by an angle of θ.

Conversion From Parallel To Series Form

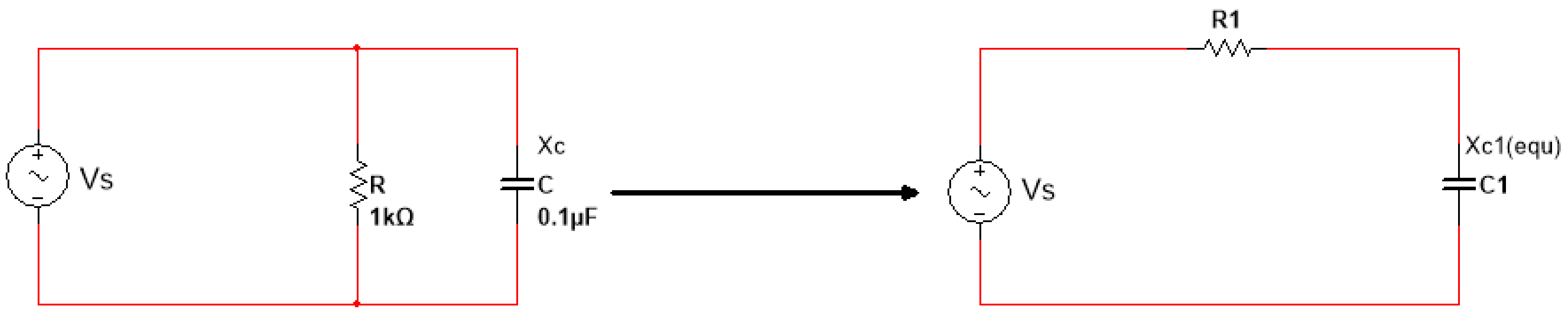
- A parallel RC circuit which has a Z_T with a phase angle θ, can be converted into a series R_1C_1 circuit by using the following formulas:
- $Z_T = \frac{R \times X_C}{\sqrt{R^2+Xc^2}}$ is the total impedance of the parallel circuit.
- $\theta = \tan^{-1}(\frac{R}{Xc})$ is the phase angle between the total current and the total voltage.

- $R_1 = Z_T \times Cos\ (\theta)$
- $X_{c1(equ)} = Z_T \times Sin\ (\theta)$
- Such equations can be used to make the analysis of a series-Parallel RC circuit easier.



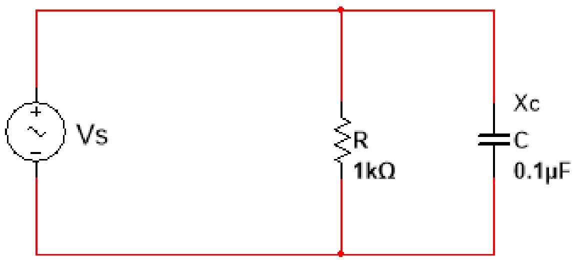
Parallel to Series Conversion Example

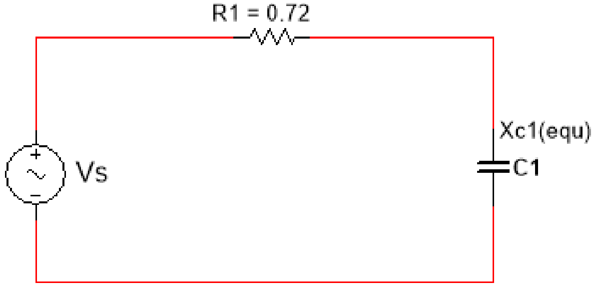
- Convert the following circuit into its series equivalent.



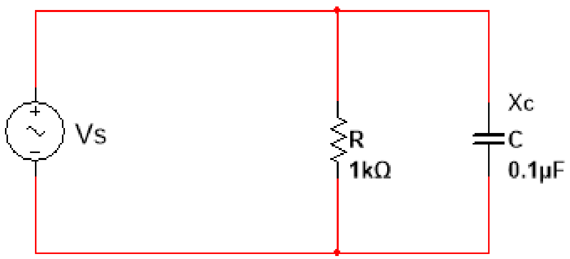
Parallel to Series Conversion Example

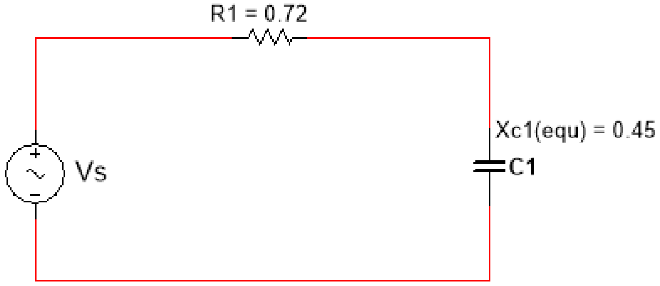
- Convert the following circuit into its series equivalent Solution:





- From a previous example, the total impedance and its phase angle were calculated as:
  - $|Z_T| = \frac{R \times X_C}{\sqrt{R^2+Xc^2}} = \frac{1 \times 1.59}{\sqrt{1^2+1.59^2}} = 0.846$.
  - $\theta = \tan^{-1}(-\frac{R}{X_C}) = \tan^{-1}(-\frac{1}{1.59}) = -32.16\ ^o$.
- Therefore:
- $R_1 = Z_T \times Cos\ (\theta) = 0.846 \times Cos\ (-32.16) = 0.72$
- $Xc_{1(equ)} = Z_T \times Sin\ (\theta) = 0.846 \times Sin(-32.16) = -0.45$





RC Circuits Analysis Main Concepts

- Any component in the RC circuit under AC can be thought of as a complex number.

- A resistance is a real component designated as R, but it can be thought of as a complex number with a zero imaginary part such as: $Z = R + j0$.

- A capacitor is an imaginary component designated as jX_C, but it can be thought of as complex number with a zero real part such as $Z = 0 \ - jX_C$.

- In fact, an RC circuit under AC can be completely analyzed with complex number arithmetic and then each component whether an impedance, current, voltage, power or energy can be converted back to the time domain.

- All circuit analysis rules and laws apply the same, such as ohm’s law, KVL, KCL, voltage division, current division etc…
- The symbol “||” will be used to indicate a parallel combination.

Series-Parallel RC Circuits Using Complex Numbers Arithmetic Under AC Example

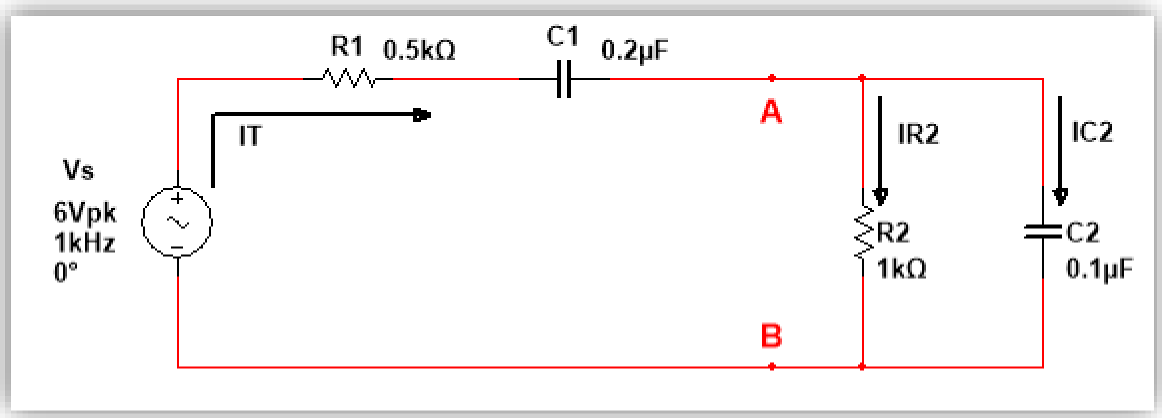
- For the following circuit find:

  a) X_{C1}, X_{C2}.

  b) Z_T seen by the voltage source V_s .

  c) $I_T, I_{R1}, I_{C1}, V_{R1}, V_{C1}, I_{R2}, I_{C2}, V_{R2}$ and V_{C2} and the phase difference between V_S & I_T.

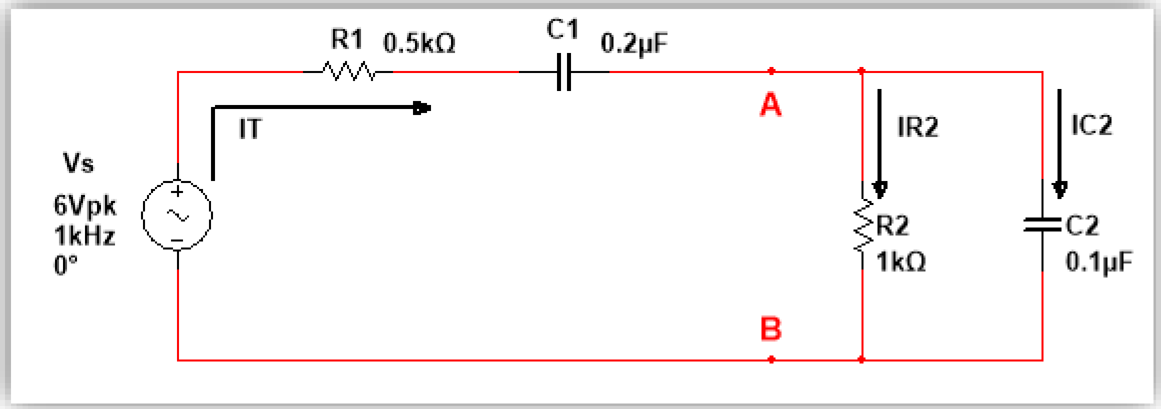
  d) Verify that the KVL applies by verifying that the sum of V_{R1} , V_{C1} and V_{R2} or V_{C2} is equal to the total voltage V_s



Series-Parallel RC Circuits Using Complex Numbers Arithmetic Under AC Example

a) X_{C1}, X_{C2}.

- $X_{C1} = \dfrac{1}{2\pi FC1} = \dfrac{1}{2\pi \times 1\ KHz \times 0.2\ \mu F} = 0.795\ K\Omega.$
- $X_{C2} = \dfrac{1}{2\pi FC} = \dfrac{1}{2\pi \times 1\ KHz \times 0.1\ \mu F} = 1.59\ K\Omega.$



Series-Parallel RC Circuits Using Complex Numbers Arithmetic Under AC Example Continued

b) Z_T seen by the voltage source V_s :

- Z_T is the series combination of $R1$, X_{C1} and $R2||X_{C2}$.

Therefore:

- $Z_T = R1 + X_{C1} + R2||X_{C2}$
- $Z_T = 0.5\ K\Omega\ - j\ 0.795\ K\Omega + 1\ K\Omega\ ||\ (-j1.59\ K\Omega)$

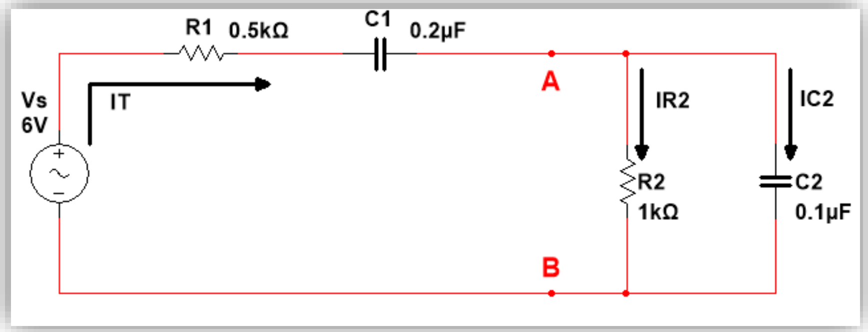
- Recall that when there are 2 elements in parallel, their equivalent will be their product divided by their sum. Therefore:
- $Z_T = 0.5\ K\Omega\ - j\ 0.795\ K\Omega + (\frac{1K \times -j1.59}{1K\ -j1.59})$

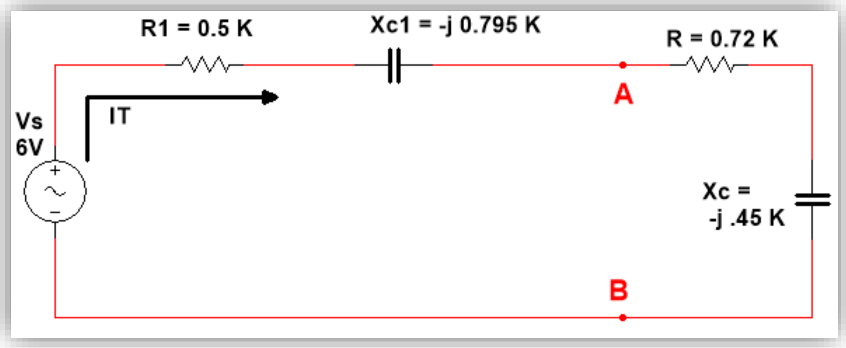
- Using complex number arithmetic such as multiplying by the conjugate results in:
- $Z_T = 0.5\ K\Omega\ - j\ 0.795\ K\Omega + (0.72\ - j0.45)$
- $\boldsymbol{Z_T = 1.22\ K\Omega\ - j\ 1.245}$

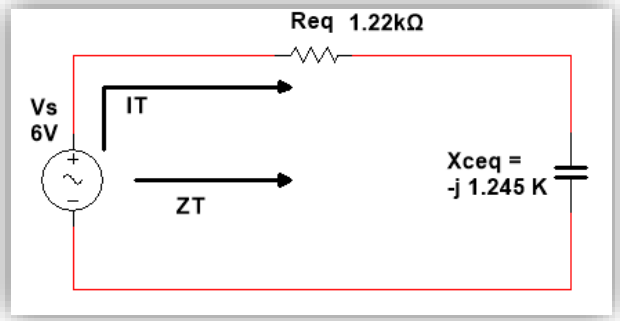
- Z_T expressed in polar format is $\boldsymbol{Z_T}$ **= 3.03 KΩ ∠ -45.58°**

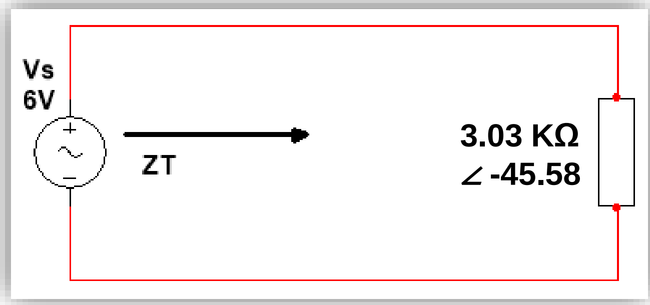
Series-Parallel RC Circuits Using Complex Numbers Arithmetic Under AC Example Continued

b) Z_T seen by the voltage source V_s Continued









Series-Parallel RC Circuits Using Complex Numbers Arithmetic Under AC Example Continued

c) $I_T, I_{R1}, I_{C1}, V_{R1}, V_{C1}, I_{R2}, I_{C2}, V_{R2}$ and V_{C2} and the phase difference between V_S & I_T

- $I_T = \frac{V_T}{Z_T} = \frac{6V}{1.22\ K\Omega - j\ 1.245}$. Using Complex numbers arithmetic and multiplying by the conjugate, such as:
  - $I_T = \frac{6V}{1.22\ K\Omega - j\ 1.245} \times \frac{1.22\ K\Omega + j\ 1.245}{1.22\ K\Omega + j\ 1.245}$
  - In rectangular format:
  - $I_T = (2.41 + j\ 2.45)$ mA
  - In polar format:

  $I_T = 3.43$ mA $\angle +45.47$

- The values of I_{R1} and I_{C1} are given by: $I_{R1} = I_{C1} = I_T = (12.41 + j\ 2.45)$ mA =3.43 mA $\angle$+45.47

Series-Parallel RC Circuits Using Complex Numbers Arithmetic Under AC Example Continued

c) $V_{R1}, V_{C1}, I_{R2}, I_{C2}, V_{R2}$ and V_{C2} and the phase difference between V_S & I_T

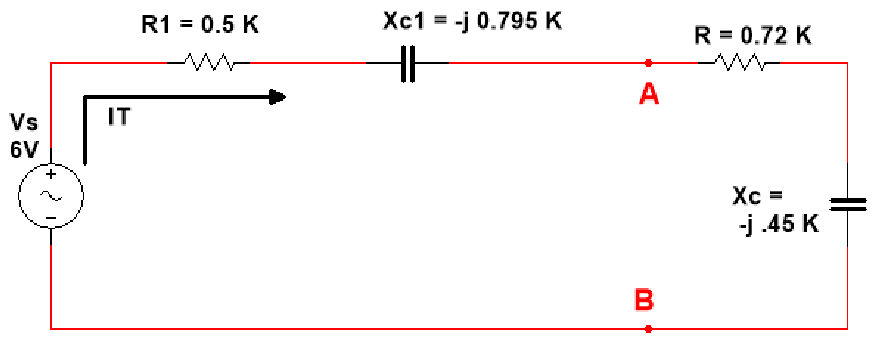
- $V_{R1} = I_{R1} \times R_1 = (2.41 + j\ 2.45)$ mA $\times\ 0.5\ K\Omega$
  - In rectangular format:
  - $V_{R1} = 1.205 + j1.225$
  - In Polar Format:
  - $V_{R1} = 1.715 \angle{+45.47}$
- $V_{C1} = I_{C1} \times X_{C1} = (2.41 + j\ 2.45)$ mA $\times\ (-j0.795\ K\Omega\ )$
  - In rectangular format:
  - $V_{C1} = (1.95\ - j1.91)$ V
  - In Polar Format:
  - $V_{C1} = 2.72 \angle{-44.53}$ V

Series-Parallel RC Circuits Using Complex Numbers Arithmetic Under AC Example Continued

c) I_{R2}, I_{C2}, V_{R2} and V_{C2} and the phase difference between V_S & I_T

- The values of the currents through R_2 and C_2 can be found either by:
    - Using the current division rule where the total current I_T is divided between and R_2 and $X_{C2.}$
    - Or:
    - By finding the voltage between points A & B, V_{AB} which is effectively the voltage across the parallel combination of R_2 and $X_{C2.}$. Once that is found, Ohm's law can be used to find the currents. The latter method will be used.

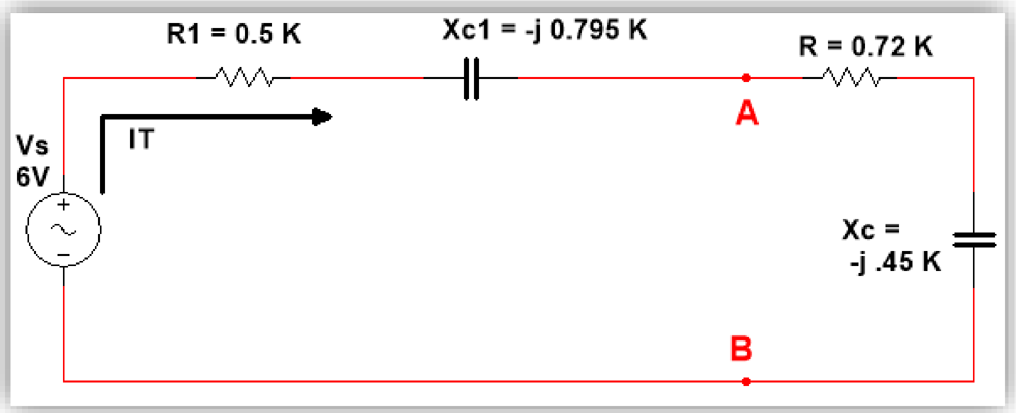
- Using ohm's law, the voltage across the parallel combination of R_2 and X_{C2} is calculated by multiplying the total current I_T by the impedance between points A & B. The impedance between points A & B is shown in the following and it is equal to the series combination of $R = 0.72\ K\Omega$ and $X_{C2} = 0.45\ K\Omega$ and that is $(0.72\ - j\ 0.45)K\Omega$.



c) V_{R2} and V_{C2} and the phase difference between V_S & I_T

- $V_{AB} = I_T \times (R + X_C) = (2.41 + j\,2.45)\text{ mA} \times (0.72\,K\Omega - j0.45\,K\Omega)$
- $V_{AB} = 1.73 - j1.0845 + j1.76 + 1.10 = 2.83 + j0.6755$ in rectangular format.
- $V_{AB} = 2.91\angle\, 13.42\,^{\circ}\, V$ in polar format.

- Since the voltage between point A & B is the same as the voltage across R2 and C2, then:
- $V_{AB} = V_{R2} = V_{C2} = 2.83 + j0.6755 = 2.91\angle\, 13.42\,^{\circ}\, V.$

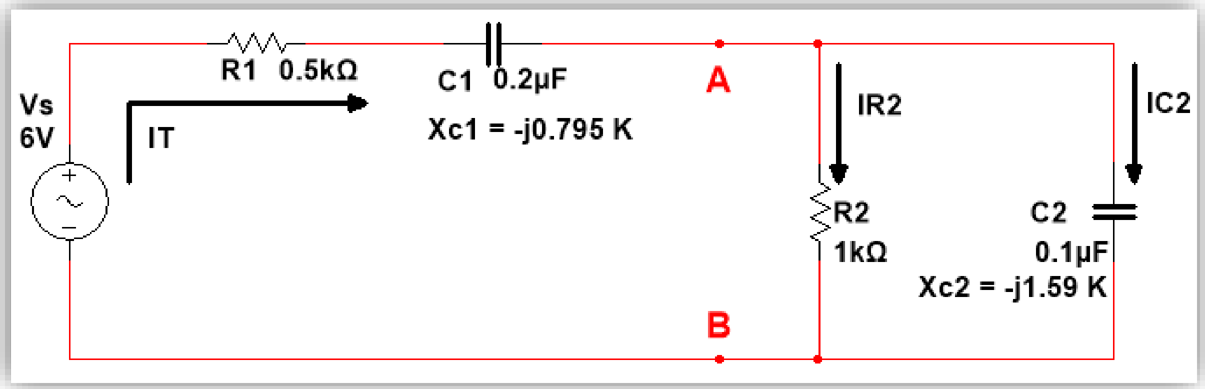


Series-Parallel RC Circuits Using Complex Numbers Arithmetic Under AC Example Continued

c) I_{R2}, I_{C2}, and the phase difference between V_S & I_T

- $I_{R2} = \frac{V_{R2}}{R2} = \frac{2.83 + j0.6755}{1\ K\Omega} = (2.83 + j0.6755)\ mA = 2.91\angle\ 13.42^\circ\ mA.$
- $I_{C2} = \frac{V_{C2}}{X_{C2}} = \frac{2.91\angle\ 13.42^\circ\ V}{-j1.59\ K\Omega} = \frac{2.91\angle\ 13.42^\circ\ V}{1.59\ K\Omega\angle -90^\circ} = 1.83\angle\ 103.42^\circ\ mA.$

- KCL can be verified by showing that $I_T = I_{R2} + I_{C2}$ using the following formula for a total current entering a parallel combination:
- $I_T = \sqrt{{I_R}^2 + {I_C}^2} = \sqrt{2.91^2 + 1.83^2} = 3.43\ mA$ which is the magnitude of the total current I_T.
- The phase difference between the voltage source V_S and the total current I_T is the phase angle of I_T itself which is 45.54° with the total current leading the total voltage source V_S.



d) Verify that the KVL applies by verifying that the sum of V_{R1}, V_{C1} and V_{R2} or V_{C2} is equal to the total voltage V_s .

The voltages in the rectangular format of the voltages across all the elements in the following circuit:

- $V_{R1} = (1.205 + j1.225)\ V$
- $V_{C1} = (1.95\ - j1.91)\ V$
- $V_{AB} = V_{R2} = V_{C2} = (2.83 + j0.6755) V$
- $V_S = V_{R1} + V_{C1} + V_{AB} = (1.205 + 1.95 + 2.83) + j(1.225\ - 1.91 + 0.6755) = 5.99\ - j0.0095\ \approx 6V$
- This is the value of the voltage source V_S.