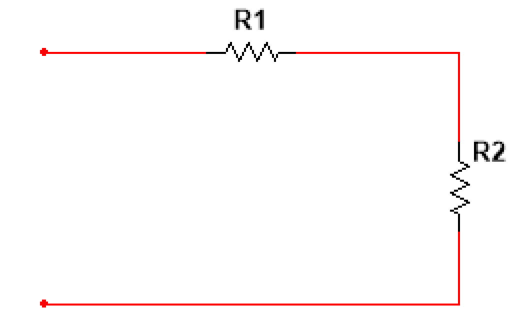
Paramount Electronics Services

Series Resistive Circuits

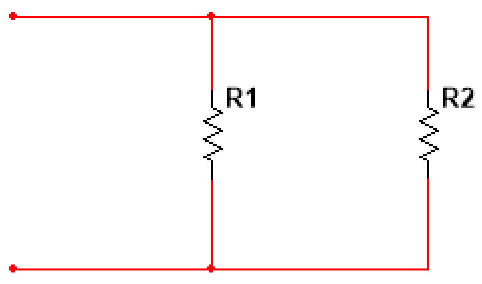
Series Circuits & Parallel Circuit

- Most circuits, no matter how complex, contain components in series and/or in parallel or a combination of both.

- Sometimes however, the circuit can neither be classified as series nor parallel. This is when special circuit analysis techniques are employed.



Series Branch with two resistors in series

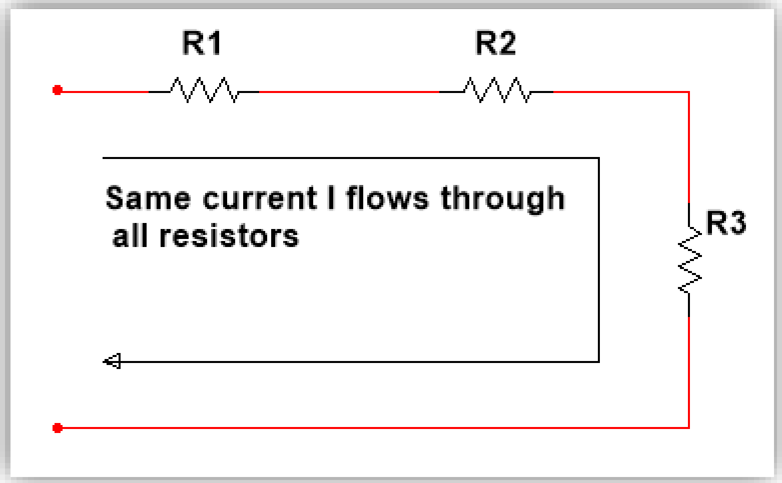


Two resistors in parallel

Series Circuit Characteristics And Total Resistance

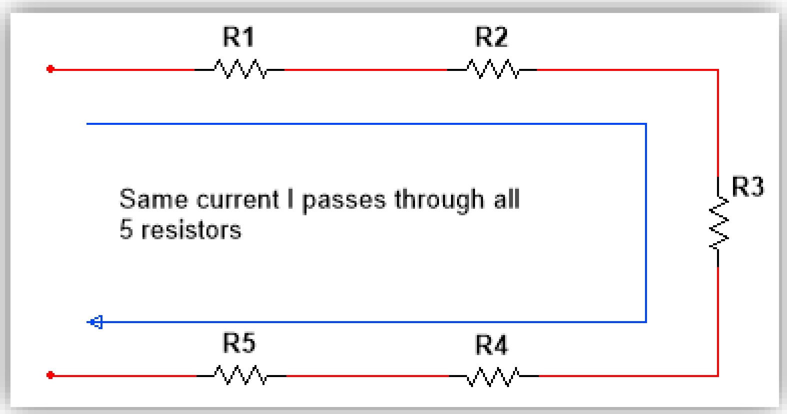
- A series resistive circuit is a network of resistances in series.

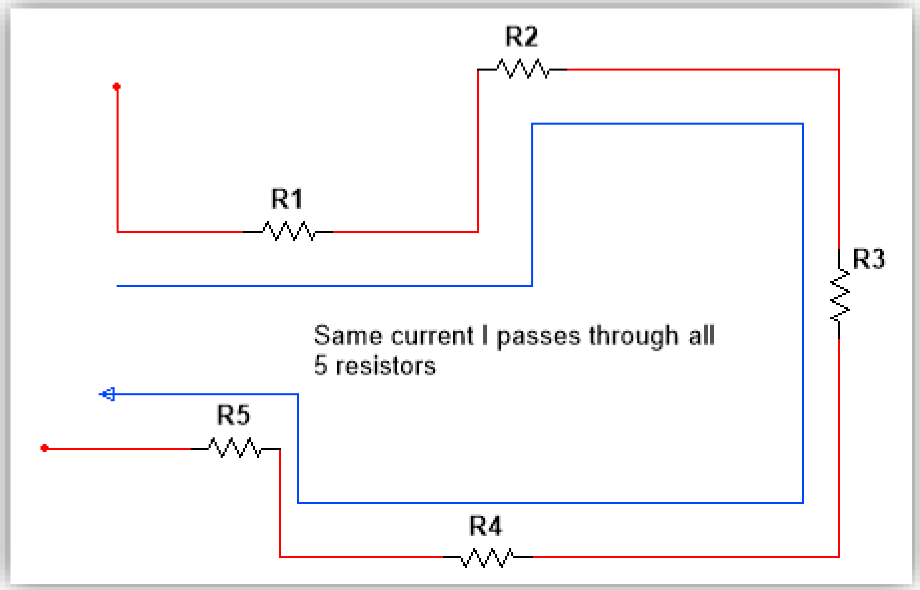
- Resistances R_1 R_2 and R_3 are in series, and they have the same current passing through them. The following is a series network of 3 resistances.



Series Circuit Characteristics And Total Resistance

A ***series circuit*** is one that has the same current passing through all the resistances. Series resistances are recognized as such by the fact that the same current passes through all of them.





The above two circuits represent the same series combination of R1, R2, R3, R4 & R5.

Series Circuit Characteristics And Total Resistance

- All circuits regardless of their structure will always read a certain resistance between any two points in the circuit.

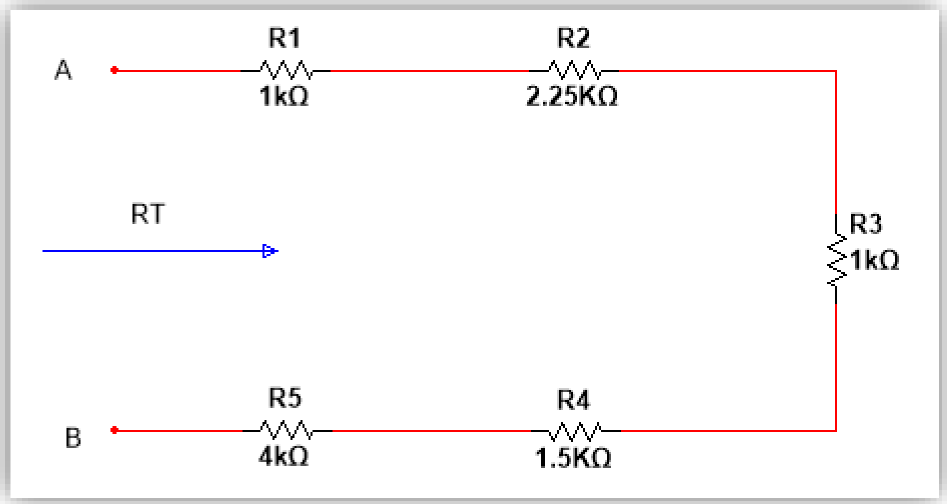
- When a source is connected to a series resistor combination, that voltage source will see a resistance value which is the equivalent resistance of the series combination.

- For a series combination of resistors R_1, R_2, R_3, … R_n the equivalent resistance is normally called the total resistance R_T and it is equal to the sum of all individual resistance values.

- $R_T = R_1 + R_2 + R_3 + \ldots + R_n$

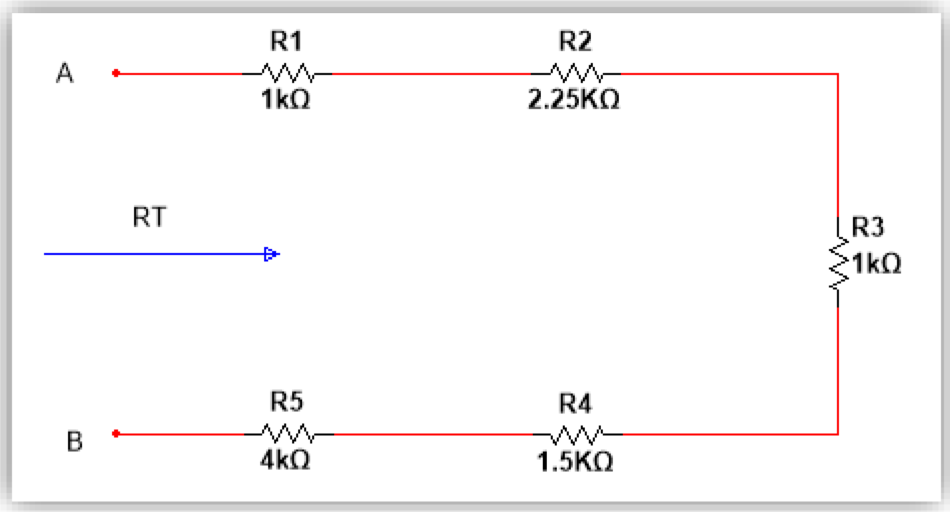
Series Circuit Example1

- For the following circuit, what is the total resistance seen between points A & B.
- If a voltage source of 19.5 V is applied to the circuit, then what is the current flow through the circuit.



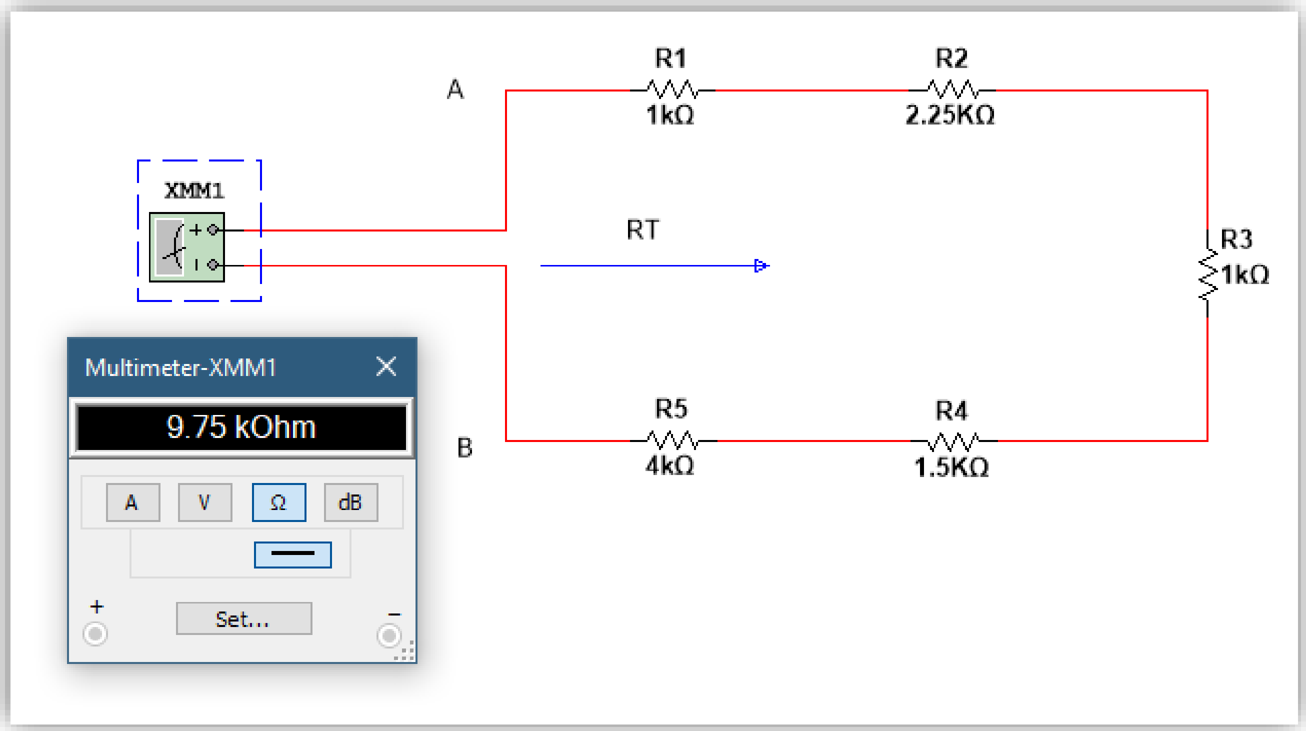
Series Circuit Example1Continued

- For the following circuit, what is the total resistance seen between points A & B.
- This is a series combination of resistances, so the total resistance between points A & B is given by:
- $R_T = R_1 + R_2 + R_3 + R_4 + R_5$
- $R_T = 1K + 2.25K + 1K + 1.5K + 4K$
- $R_T = 9.75K$



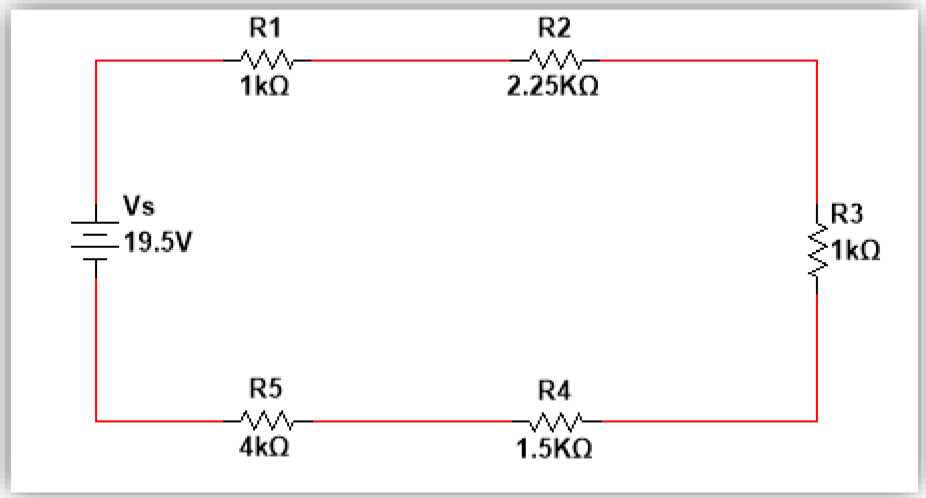
Series Circuit Example1 Continued

- For the following circuit, what is the total resistance seen between points A & B.
- Using the Multi-Simulation tool and using a multimeter set to read resistance values, it is shown that the total resistance between points A & B is equal to 9.75K ohms.



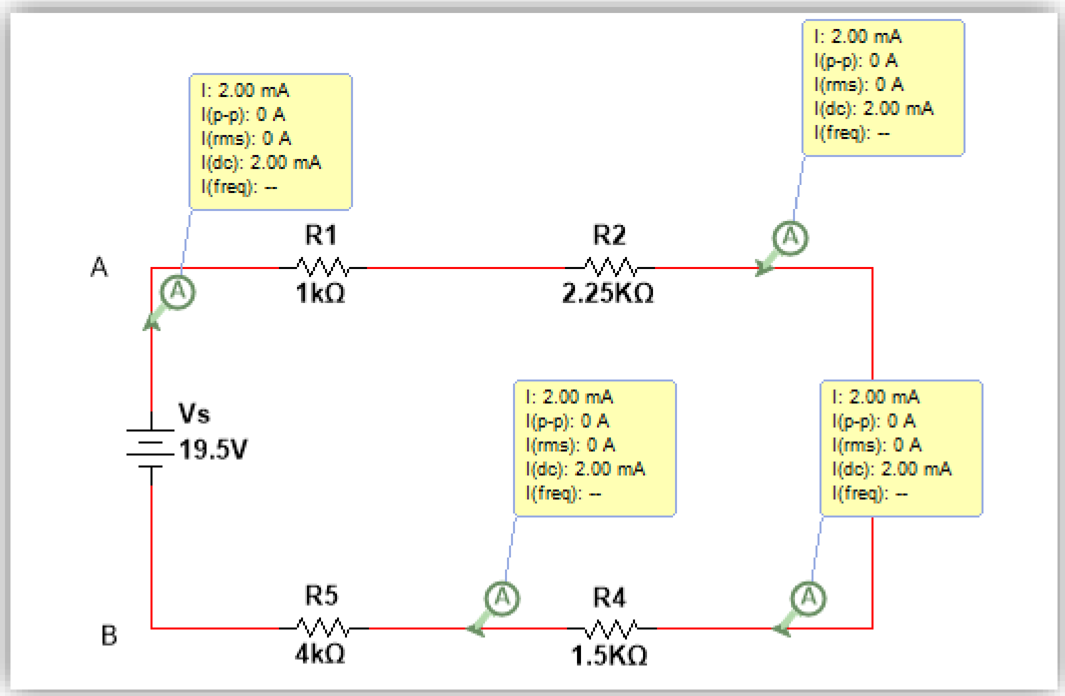
Series Circuit Example1 Continued

- If a voltage source of 19.5 V is applied to the circuit, then what is the current flow through the circuit?
- Because there is only one path, the current everywhere is the same.
- Using Ohm's Law, the current that is sourced by the voltage source is equal to the voltage source value divided by the total resistance R_T that the voltage source sees.
- $R_T = R_1 + R_2 + R_3 + R_4 + R_5$
- R_T = 1K + 2.25K + 1K + 1.5K + 4K = 9.75K
- $I_T = V_T / R_T = V_S / R_T$ = 19.5V / 9.75KΩ = 2 mA



Series Circuit Example1 Continued

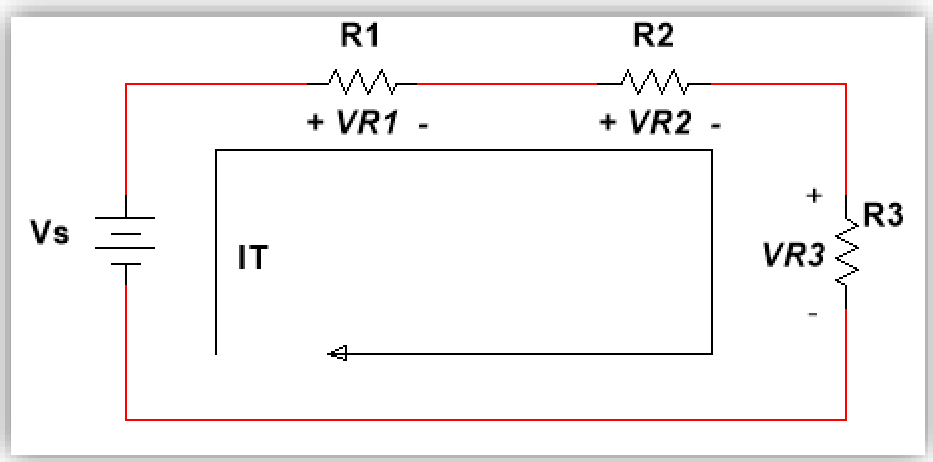
- If a voltage source of 19.5 V is applied to the circuit, then what is the current flow through the circuit.
- Using the Multi-Simulation tool and using current probes, it is shown that the current is the same through out the series circuit. In this case it is equal to 2 mA (2 milli Amperes).



Kirchhoff's Voltage Law, KVL

- Kirchhoff's Voltage Law (KVL) states that the sum of all voltages in a loop is equal to zero.

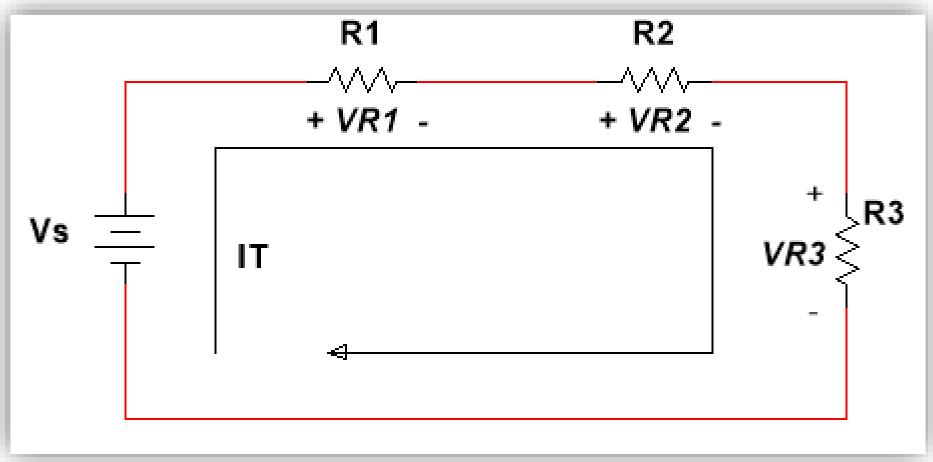
- If the voltages are tracked around the loop a plus sign is placed in front of the voltage if the + is encountered first and a minus sign is placed in front of the voltage if the – is encountered first, then:
  - $-V_s + V_{R1} + V_{R2} + V_{R3} = 0$



Kirchhoff's Voltage Law, KVL

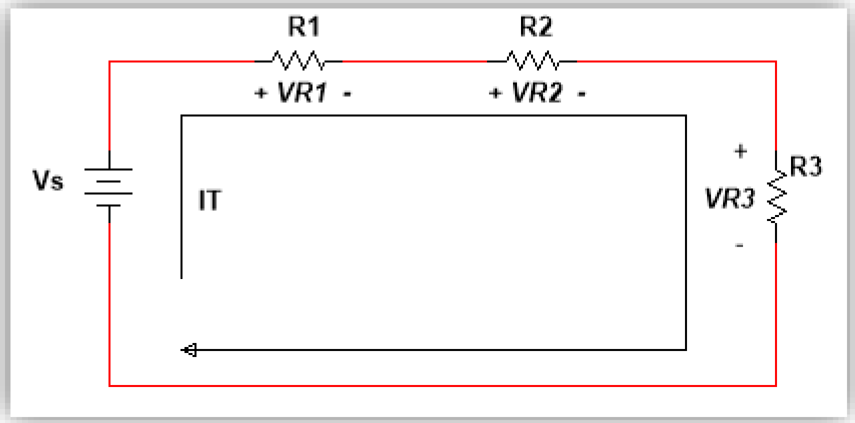
- KVL, stated otherwise, in a loop, the applied voltage is distributed among the various elements of the loop (regardless of what elements are used such as resistors, capacitors, inductors etc.…).

- So, for the following circuit:
  - $V_s = V_{R1} + V_{R2} + V_{R3}$



Kirchhoff's Voltage Law Example1

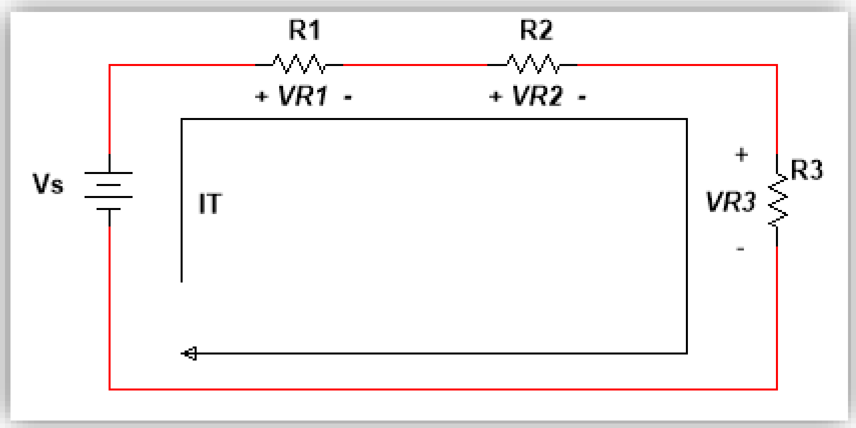
- For the following circuit:
- a) If V_{R1}, V_{R2} and V_{R3} are equal to 1.5V, 2V and 4.5V respectively, then what is the value of the voltage source V_s?
- b) If V_s, V_{R2} and V_{R3} are equal to 10V, 2V and 4.5V respectively, then what is the value of the voltage V_{R1}?



Kirchhoff's Voltage Law Example1

- a) If V_{R1}, V_{R2} and V_{R3} are equal to 1.5V, 2V and 4.5V respectively, then what is the value of the voltage source V_s?

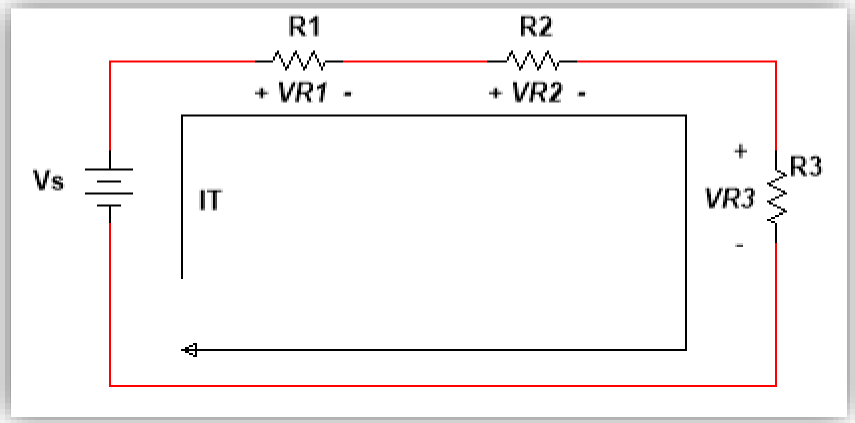
- According to KVL, the voltage source is distributed across the various elements in the loop, therefore:
- $V_s = V_{R1} + V_{R2} + V_{R3}$
- V_s = 1.5V + 2V + 4.5V = 8V



Kirchhoff's Voltage Law Example1

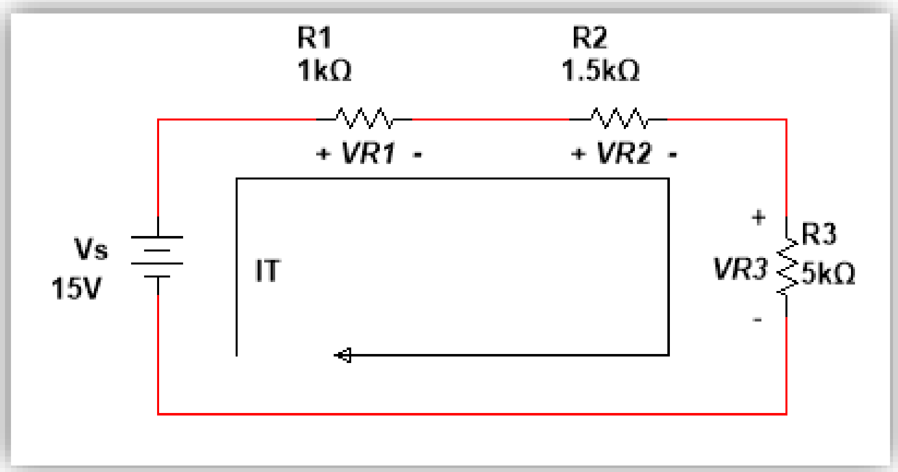
- b) If V_s, V_{R2} and V_{R3} are equal to 10V, 2V and 4.5V respectively, then what is the value of the voltage source V_{R1}?

- According to KVL, the voltage source is distributed across the various elements in the loop, therefore:
- $V_s = V_{R1} + V_{R2} + V_{R3}$
- $V_{R1} = V_s - (V_{R2} + V_{R3})$
- $V_{R1} = 10V – (2V + 4.5V)$
- $V_{R1} = 3.5V$



Kirchhoff's Voltage Law Example2

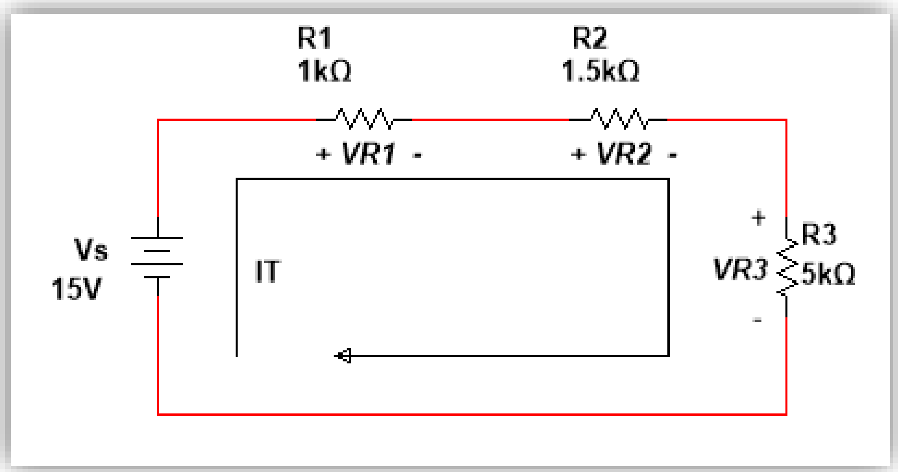
- For the following circuit, find:
  a) The total resistance seen by the source voltage.
  b) The total current I_T passing through the series circuit.
  c) The values of the voltage V_{R1}, V_{R2}, V_{R3}
  d) The power delivered by the source and the powers consumed by the resistors.
  e) Verify that the power delivered by the voltage source is equal to the sum of the powers consumed by the resistors in the circuit.



Kirchhoff’s Voltage Law Example2

a) The total resistance seen by the source voltage.

- Since this is a series circuit, the total resistance seen by the source is the sum of all resistances:
  - $R_T = R_1 + R_2 + R_3 = 1K + 1.5K + 5K = 7.5\ K\Omega$

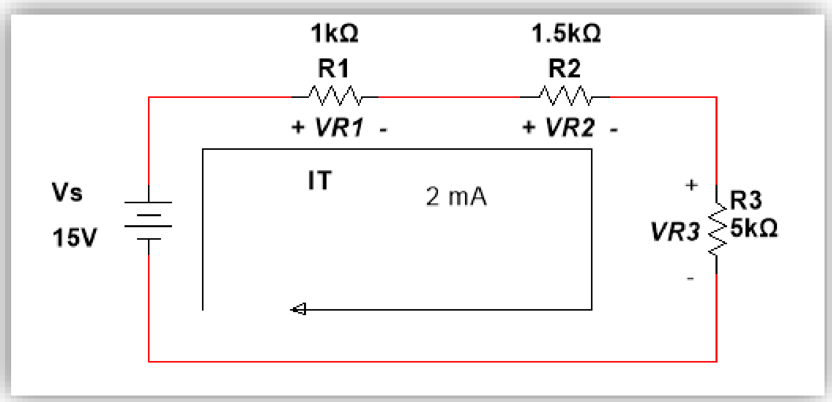


Kirchhoff’s Voltage Law Example2

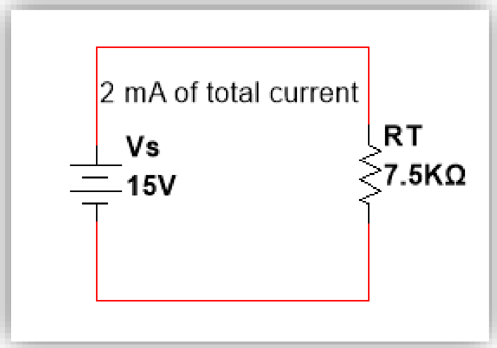
b) The total current I_T passing through the series circuit.

- Since the applied voltage is dropped across the various elements of the circuit and there is only one element which is R_T then the voltage drop across R_T is V_{RT} and it is equal to V_s =15V.

- Therefore, using ohm’s low which states that:
- $I_R = V_R / R$.
- For R_T , $I_{RT} = V_{RT} / R_T = V_s / R_T$ = (15V / 7.5 KΩ) = 2 mA.



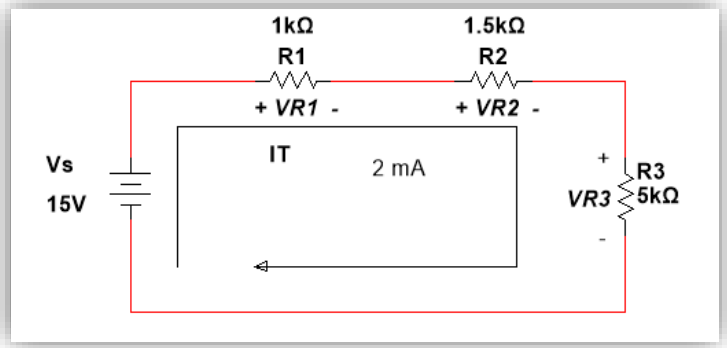
Is equivalent to:



Kirchhoff’s Voltage Law Example2

c) The values of the voltage V_{R1}, V_{R2}, V_{R3}

- The current $I_T = I_{R1} = I_{R2} = I_{R3} = 2$ mA
- According to Ohm’s Law, $V_R = I_R \times R$.
- Therefore:
  - $V_{R1} = I_{R1} \times R_1 = 2$ mA $\times$ 1 KΩ = 2 V.
  - $V_{R2} = I_{R2} \times R2 = 2$ mA $\times$ 1.5 KΩ = 3V.
  - $V_{R3} = I_{R3} \times R_3 = 2$ mA $\times$ 5KΩ = 10 V.



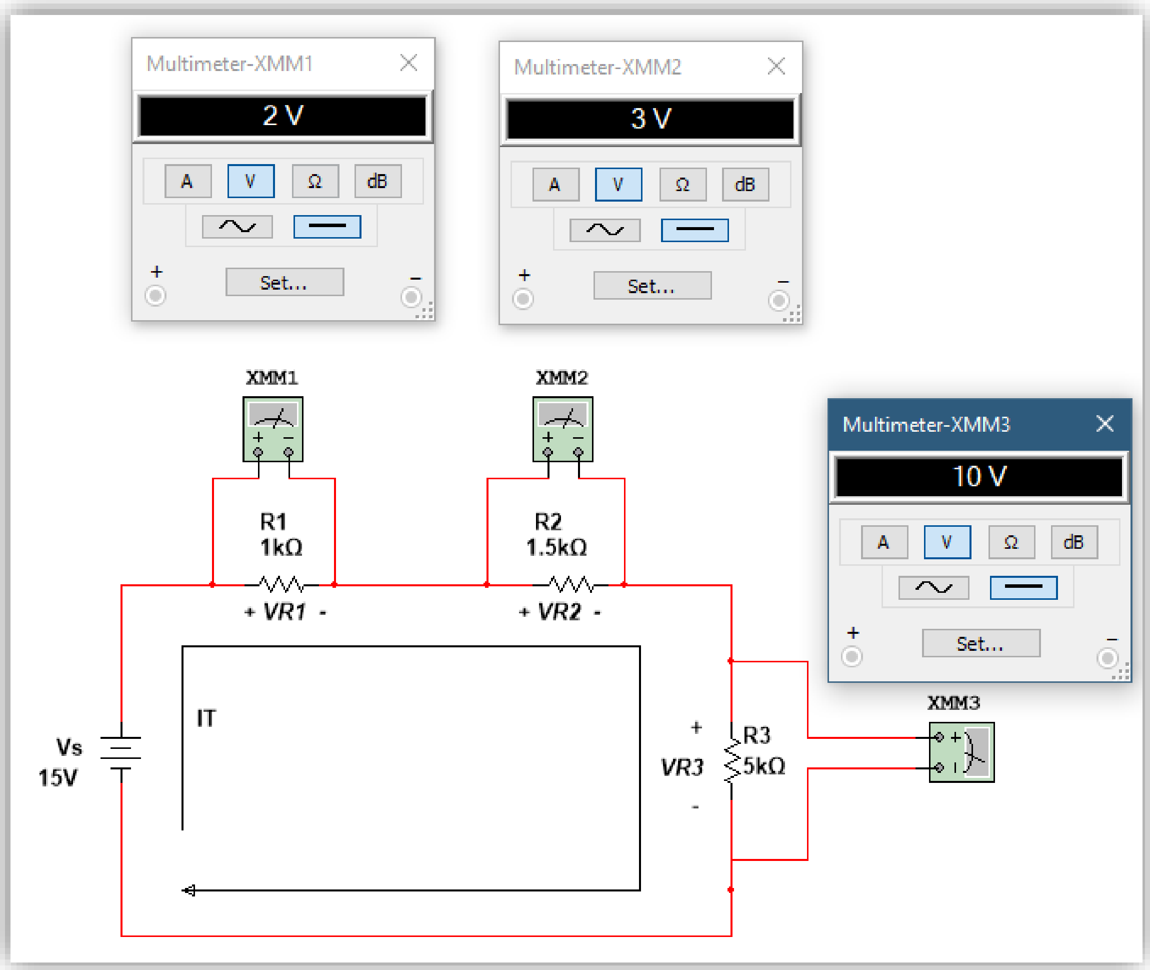
- Verifying Kirchhoff's Voltage Law:
  - $-V_s + V_{R1} + V_{R2} + V_{R3} = -15V + 2V + 3V + 10V = 0$

Or:

- $V_s = V_{R1} + V_{R2} + V_{R3}$
- 15V = 2V + 3V + 10V

Kirchhoff's Voltage Law Example2

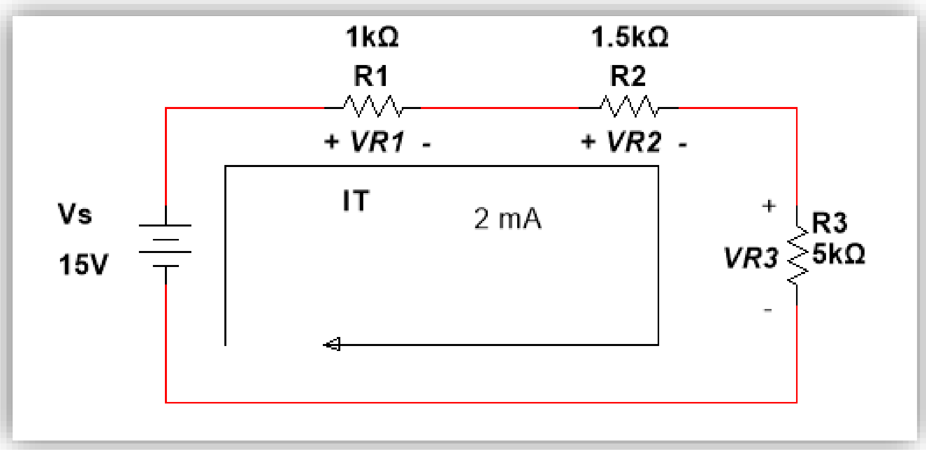
- Using the Multi-Simulation tool and using multimeters set to measure voltage, it is shown that the sum of the voltages across R1, R2 & R3 is equal to the applied voltage Vs.



Kirchhoff’s Voltage Law Example2

d) The power delivered by the source and the powers consumed by the resistors.

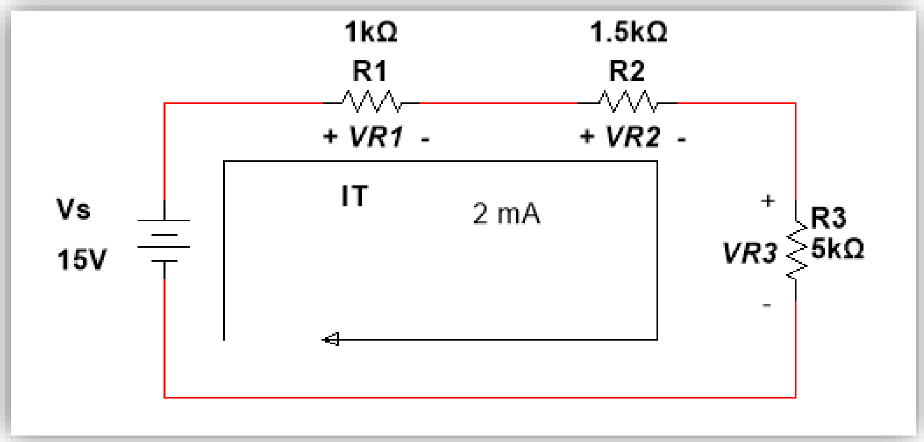
- Power delivered by a source is equal to:
  - P_{Source} = Voltage source value × Current through it.
  - $P_{Vs} = V_s \times I_T$ = 15 V × 2 mA = 30 mW.



Kirchhoff's Voltage Law Example2

d) The power delivered by the source and the powers consumed by the resistors.

- Power is always consumed by a resistance and is given by:
  - $P_R = V_R \times I_R = V_R^2$ / R = $I_R^2 \times$ R
  - $P_{R1} = V_{R1} \times I_{R1}$ = 2V × 2mA = 4 mW.
  - $P_{R2} = V_{R2}^2$ / R2 = $(3V)^2$ / 1.5 KΩ = 9 /1.5 KΩ = 6 mW
  - $P_{R3} = I_{R3}^2 \times$ R3 = $(2mA)^2 \times$ 5 KΩ = 20 mW



Kirchhoff's Voltage Law Example2

e) Verify that the power delivered by the voltage source is equal to the sum of the powers consumed by the resistors in the circuit.

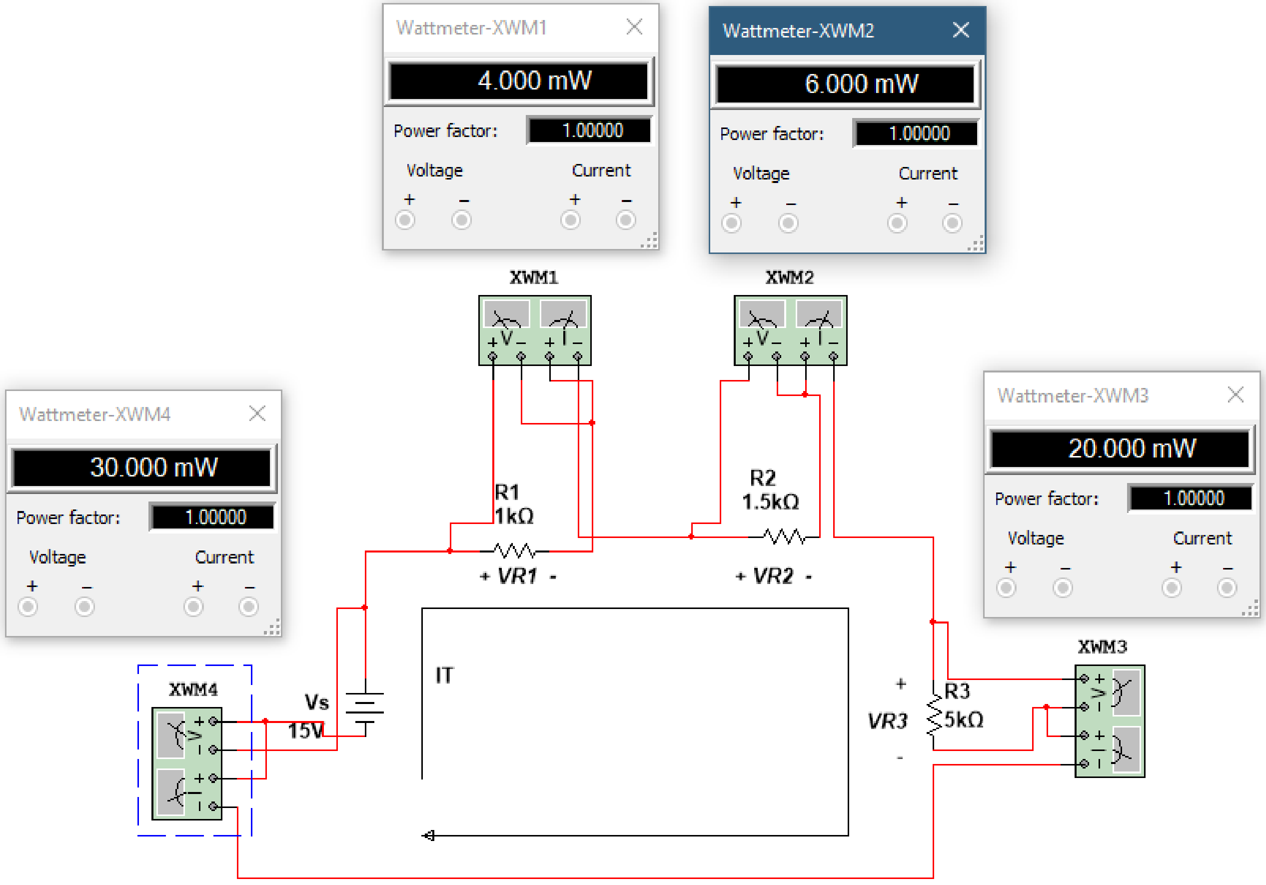
- Regardless of the circuit structure, the power delivered by the sources will always equal to the sum of individual powers consumed by the resistors. Therefore:

P_{Vs} = 30 mW.
P_{R1} = 4 mW.
P_{R2} = 6 mW.
P_{R3} = 20 mW

$P_{Vs} = P_{R1} + P_{R2} + P_{R3}$
30 mW = 4 mW + 6 mW + 20 mW = 30 mW.

Kirchhoff's Voltage Law Example2

- Using the Multi-Simulation tool and using wattmeters set to measure power, it is shown that the sum of the powers consumed by R1, R2 & R3 is equal to the power supplied by the applied voltage Vs.



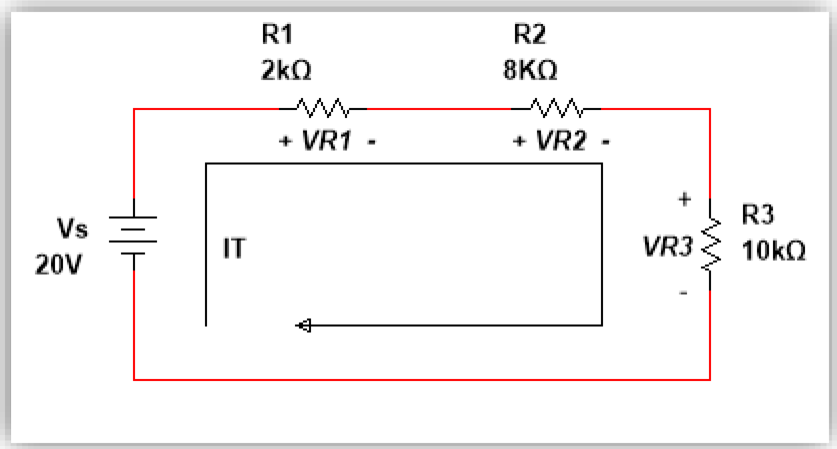
The Voltage Divider Formula

- For a series resistive circuit branch with an applied voltage V_T, the individual voltages across the resistors in the branch are calculated using the voltage divider formula which is given by:
- $V_{Rx} = V_T \times \frac{R_x}{R_T}$, Where:
  - x = 0, 1, 2, 3, …, n
  - V_T is the applied voltage to the series circuit branch.
  - R_T is the total resistance of series circuit branch
  - V_T is not necessarily the voltage source supplying the whole circuit, but it could be.
- In other words:
- $V_{R1} = V_T \times \frac{R_1}{R_T}$ is the voltage across R_1
- $V_{R2} = V_T \times \frac{R_2}{R_T}$ is the voltage across R_2
- $V_{R3} = V_T \times \frac{R_3}{R_T}$ is the voltage across R_3
- ....
- $V_{Rn} = V_T \times \frac{R_n}{R_T}$ is the voltage across R_n

- R_1, R_2 , R_3 , ……., R_n are the resistances in series circuit branch.

Voltage Division Example3

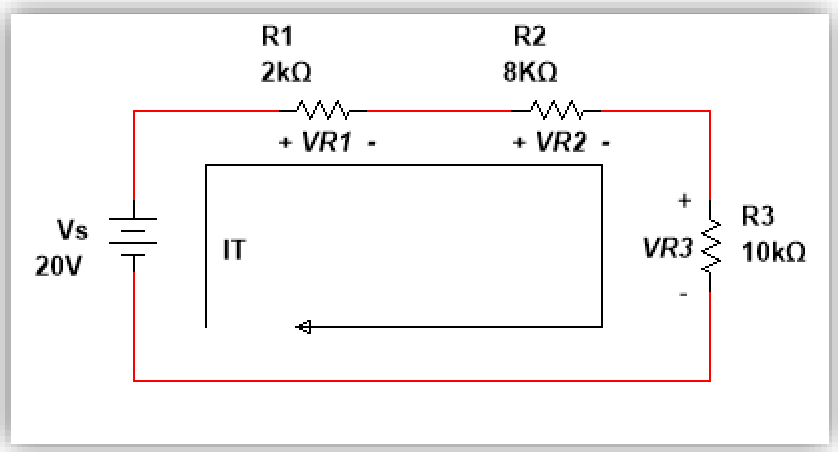
- For the following circuit, find:
  a) The total resistance seen by the source voltage.
  b) Using the Voltage Divider Formula, find the values of the voltage VR1, VR2, VR3
  c) Verify that the algebraic sum of the voltages around the loop is equal to 0 thus verifying KVL.



Voltage Division Example3

a) The total resistance seen by the source voltage.

- Since this is a series circuit, the total resistance seen by the voltage source V_s is given by the sum of all individual resistances. Therefore:
- $R_T = R_1 + R_2 + R_3 = 2\ \text{K}\Omega + 8\ \text{K}\Omega + 10\ \text{K}\Omega\ = 20\text{K}\Omega$



Voltage Division Example3

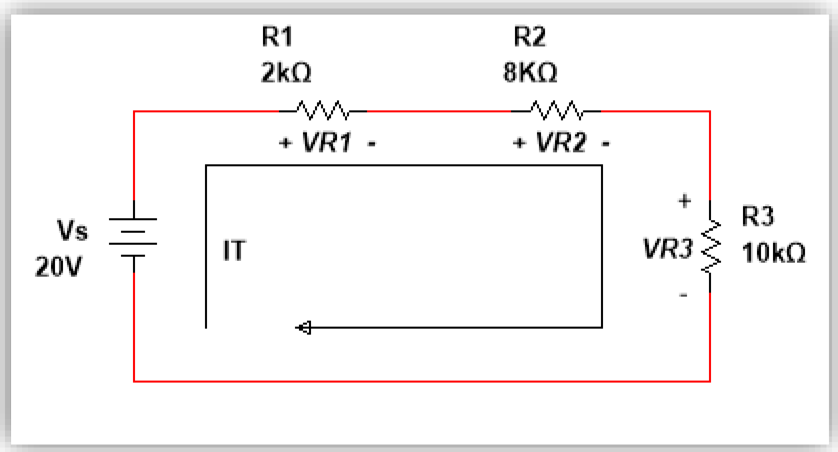
b) Using the Voltage Divider Formula, find the values of the voltage V_{R1} , V_{R2} and V_{R3}

- The general voltage divider formula which is given by:

$$V_{Rx} = V_T \times \frac{R_x}{R_T}$$

where V_T is the applied voltage V_s, therefore, V_T can be replaced by V_s in the formula.

- $V_{R1} = V_s \times \frac{R_1}{R_T} = 20\text{V} \times \frac{2K}{20K} = 2\text{V}$
- $V_{R2} = V_s \times \frac{R_2}{R_T} = 20\text{V} \times \frac{8K}{20K} = 8\text{V}$
- $V_{R3} = V_s \times \frac{R_3}{R_T} = 20\text{V} \times \frac{10K}{20K} = 10\text{V}$



- V_{R1} is the voltage across R_1
- V_{R1} is the voltage across R_2
- V_{R1} is the voltage across R_3

Voltage Division Example3

c) Verify that the algebraic sum of the voltages around the loop is equal to 0 thus verifying KVL.

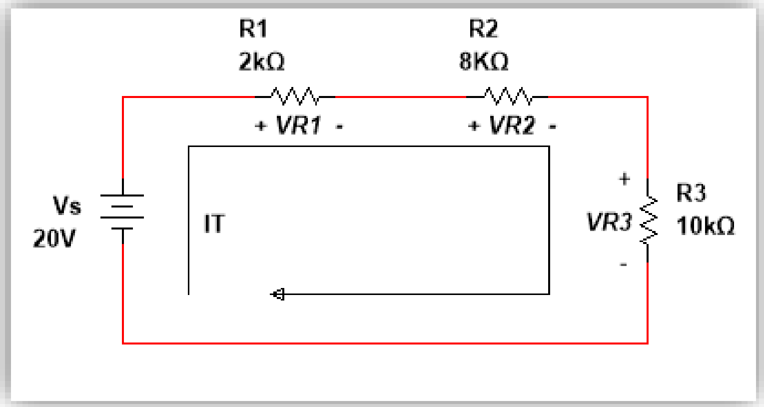
Going around the loop in a clockwise manner (could have gone in a counterclockwise manner as well) and add the voltages while taking the polarity into account, we get:

$- \; V_s + V_{R1} + V_{R2} + V_{R3}$ = -20V + 2V + 8V + 10V = 0.

Or we could use the concept that the applied voltage to the series branch is distributed across the various elements in that loop. Therefore:

$V_T = V_{R1} + V_{R2} + V_{R3}$

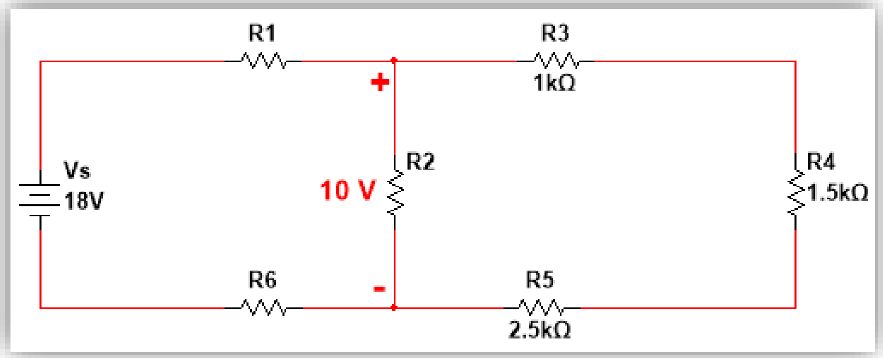
20V = 2V + 8V + 10V = 20V



Voltage Division Example4

- For the following circuit, if the voltage across R_2 is 10V, then what are the voltages across R_3 , R_4 and R_5?
- The voltage across R2, V_{R2}, is equal to 10V. This is the total voltage V_T which is across the series combination of R_3 , R_4 and R_5
- The total resistance of the series combination of R_3 , R_4 and R_5 is R_T. It is given by the sum of those resistances:
  - $R_T = R_3 + R_4 + R_5 = 1\text{K}\Omega + 1.5\text{K}\Omega + 2.5\text{K}\Omega = 5\text{K}\Omega.$

- Therefore, applying the voltage division formula $V_{Rx} = V_T \times \frac{R_x}{R_T}$:
- $V_{R3} = V_T \times \frac{R_3}{R_T} = 10V \times \frac{1K}{5K} = 2V$
- $V_{R4} = V_T \times \frac{R_4}{R_T} = 10V \times \frac{1.5K}{5K} = 3V$
- $V_{R3} = V_T \times \frac{R_5}{R_T} = 10V \times \frac{2.5K}{5K} = 5V$
- Notice that KVL applies in this case where:
  - $V_T = V_{R3} + V_{R3} + V_{R3} = 2V + 3V + 5V = 10V$



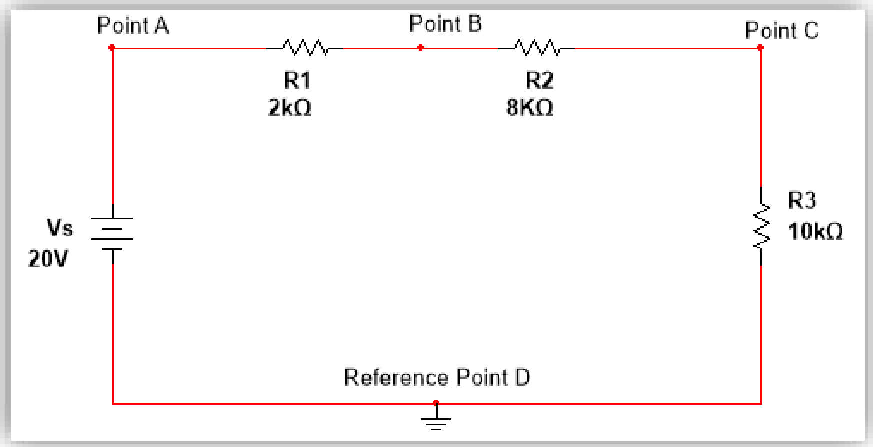
Voltage References & Voltage Notations

- Any point in a circuit can be thought of as a reference point. However, the common terminology is that the reference point is the ground point.
- The reference point is used to specify the voltage at any other point in the circuit such as V_A, V_B, V_C etc.. Rather than the voltage drop across an element in the circuit.

- The voltage V_A is the voltage between point A and the reference point.
- The voltage V_B is the voltage between point B and the reference point.
- The voltage V_C is the voltage between point C and the reference point.
- The voltage V_{AB} is the voltage between points A & B and equal to V_A - V_B .
- The voltage V_{AC} is the voltage between points A & C and equal to V_A - V_C .
- The voltage V_{BC} is the voltage between points B & C and equal to V_B - V_C .
- **The value of the voltage at the reference point is always equal to 0**.

Voltage References & Voltage Notations

- The voltage V_A is the voltage between point A and the reference point D.
- The voltage V_B is the voltage between point B and the reference point D.
- The voltage V_C is the voltage between point C and the reference point D.
- The voltage V_{AB} is the voltage between points A & B = V_A - V_B .
- The voltage V_{AC} is the voltage between points A & C = V_A - V_C .
- The voltage V_{BC} is the voltage between points B & C = V_B - V_C.
- The value of the voltage at the reference point is always equal to 0.

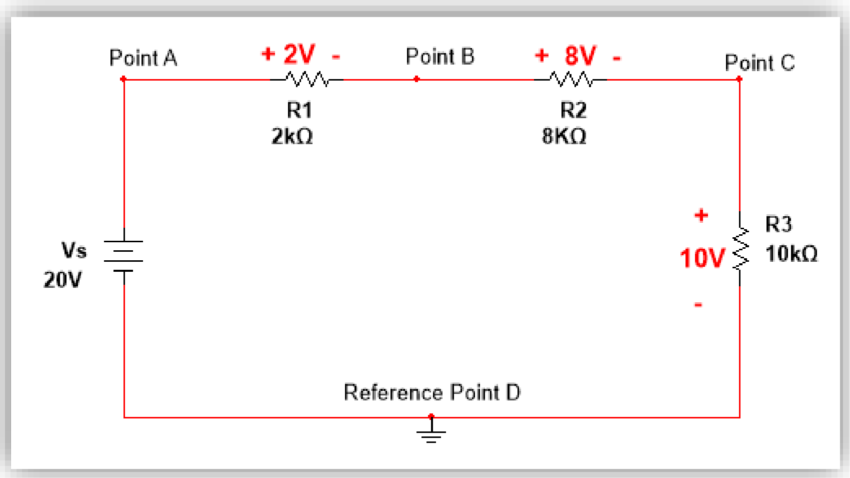


Voltage References & Voltage Notations

- The voltage V_A is the voltage between point A and the reference point D and has a value of 20V.
- The voltage V_B is the voltage between point B and the reference point D and has a value of 18V since going from point A to point B, 2V are dropped across resistance R1.
- The voltage V_C is the voltage between point C and the reference point D and has a value of 10V since going from point B to point C, 8V are dropped across R2.
- The value of the voltage at the reference point is equal to 0 since going from point C to the reference point D, 10V are dropped across R3.

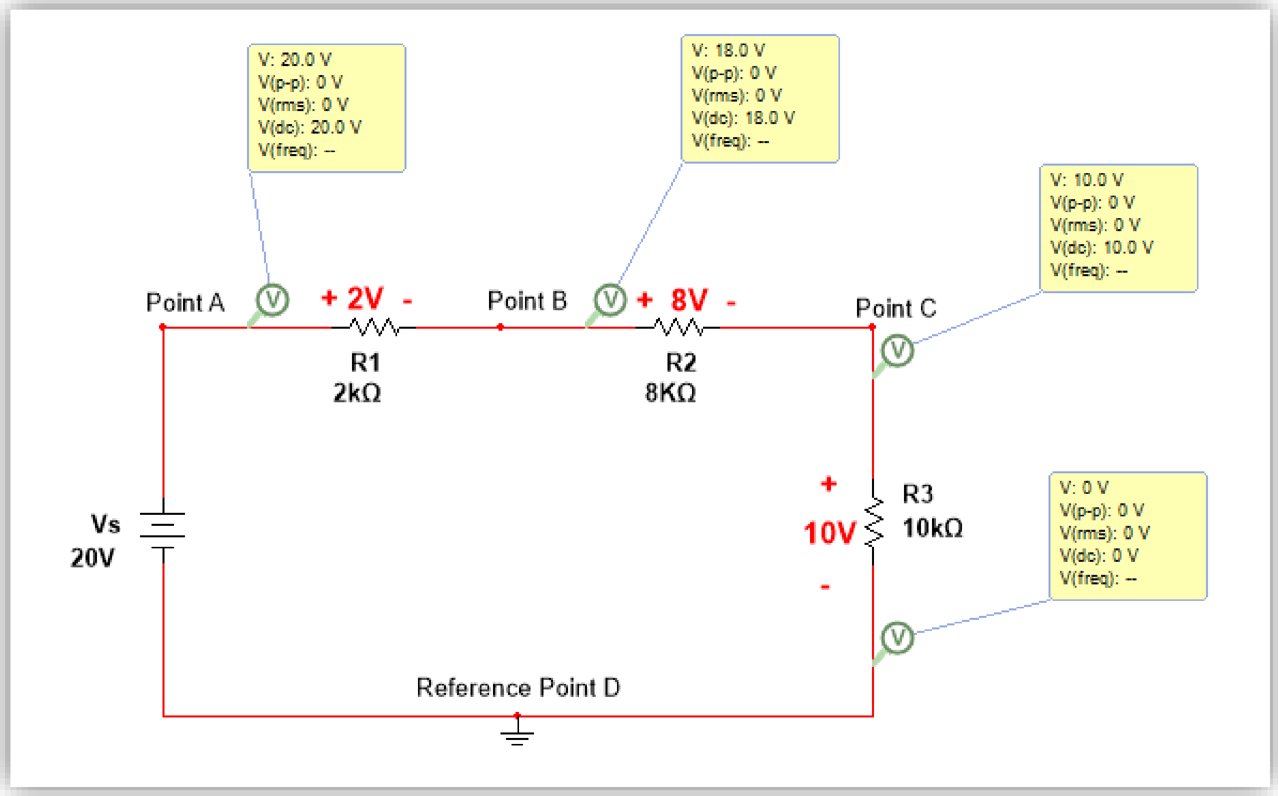
- The voltage V_{AB} = V_A - V_B = 20V – 18V = 2V, is the voltage between points A & B.
- The voltage V_{AC} = V_A - V_C = 20V – 10V = 10V, is the voltage between points A & C.
- The voltage V_{BC} = V_B - V_C = 18V – 10V = 8V, is the voltage between points B & C.

- Always V_{AB} = - V_{BA}
- Always V_{AC} = - V_{CA}
- Always V_{BC} = - V_{CB}



Voltage References & Voltage Notations

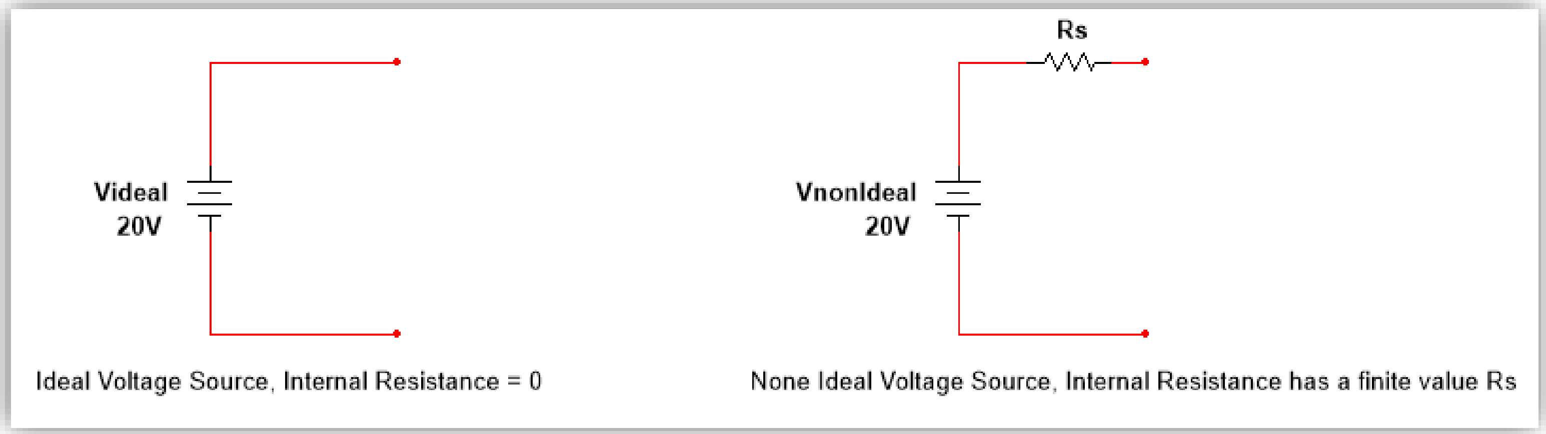
- Using the Multi-Simulation tool, it can be shown that the voltage values are as discussed earlier.



Series Circuits Practical Considerations

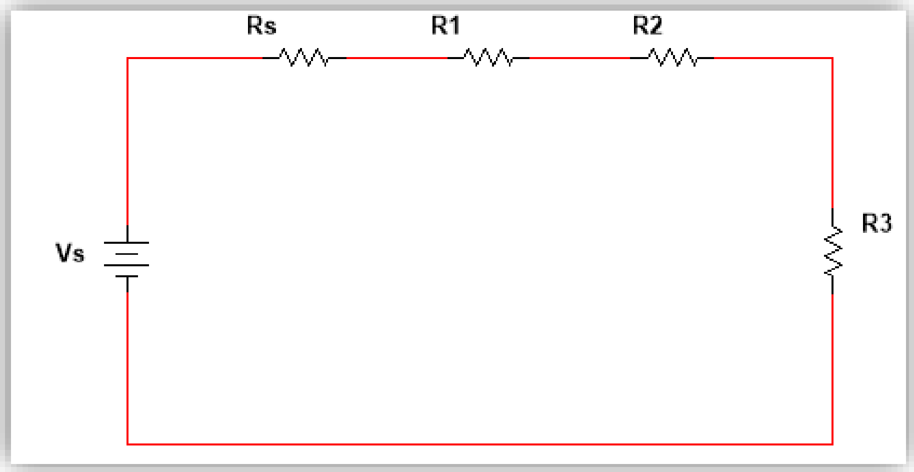
- When a voltage source is considered ideal, its internal resistance is equal to 0.

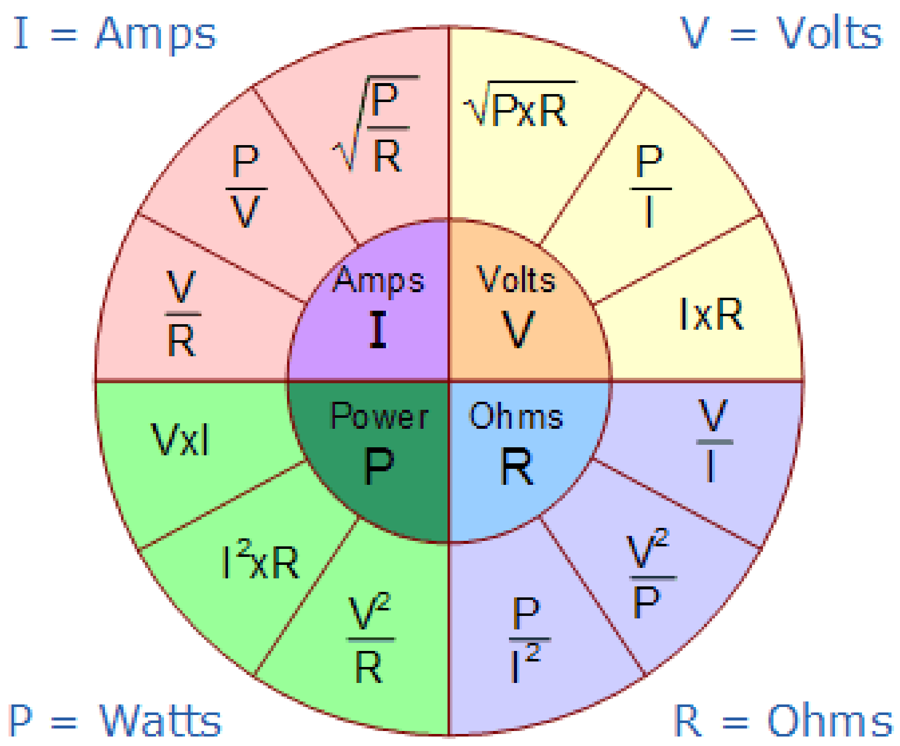
- Voltage sources are not ideal and therefore, they will have a finite internal resistance value Rs as shown below.



Series Circuits Practical Considerations

- If R_s is small compared to the remaining resistances in the series branch circuit, then it can be ignored and will not play a role in the total resistance value with a negligible error.
- If Rs however is not small compared to the remaining resistances in the series branch circuit, then it cannot be ignored and will play a role in the total resistance value where:
- $R_T = R_1 + R_2 + R_3 + R_s$
- The voltage division formula remains the same, however.
- $V_{Rx} = V_T \times \frac{R_x}{R_T}$
- Where:
  - x = 1, 2, 3, …, n
  - V_T is the applied voltage to the series circuit
  - R_T is the total resistance of series circuit branch




I = Amps
V = Volts
P = Watts
R = Ohms
Amps
I
Volts
V
Power
P
Ohms
R
$\sqrt{\frac{P}{R}}$
$\frac{P}{V}$
$\frac{V}{R}$
$\sqrt{PxR}$
$\frac{P}{I}$
IxR
$\frac{V}{I}$
$\frac{V^2}{P}$
$\frac{P}{I^2}$
VxI
I^2xR
$\frac{V^2}{R}$