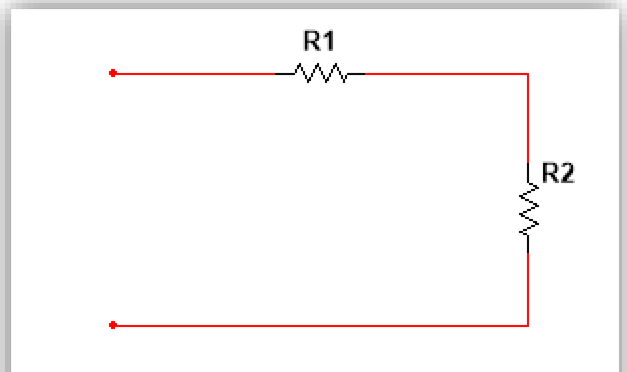
Paramount Electronics Services

Parallel Resistive Circuits

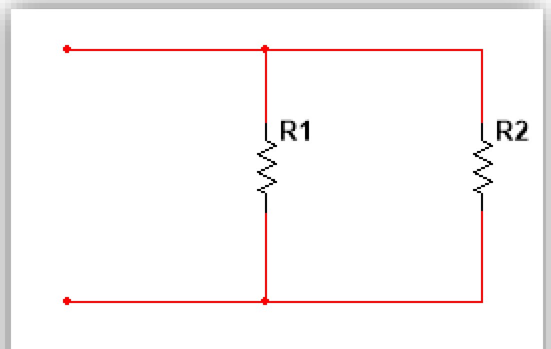
Parallel Circuit Characteristics

- Most circuits, no matter how complex, contain components in series and/or in parallel or a combination of both.

- Sometimes however, the circuit can neither be classified as series nor parallel. This is when special circuit analysis techniques are employed.



Series Branch with two resistors in series



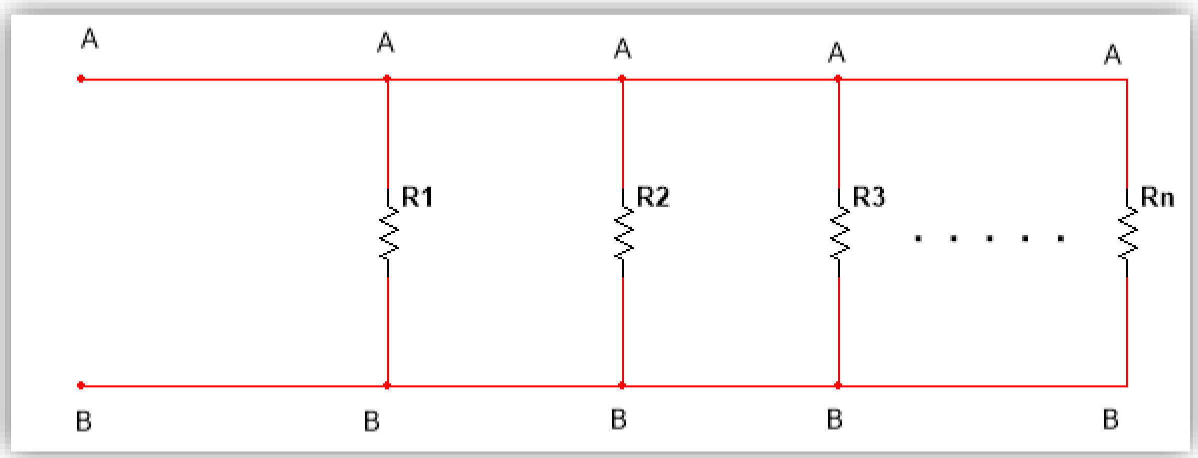
Two resistors in parallel

Parallel Circuit Characteristics

- Two or more resistors are in parallel if they share two common electrical points across their terminals.

- Two nodes are the same electrical point if there are no elements between them.

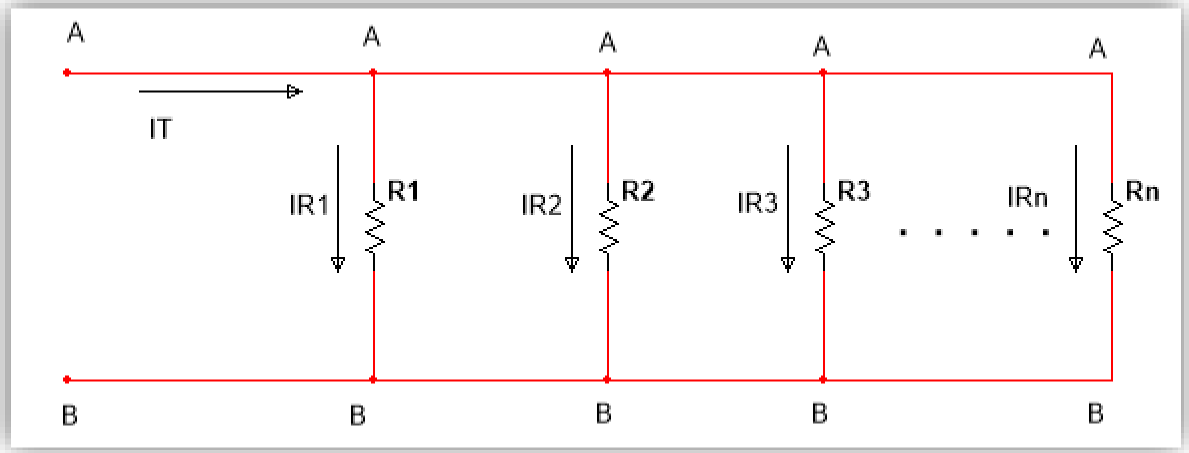
- Notice in the circuit below that the two common electrical points are A & B



Parallel Circuit Characteristics

- Parallel Circuit also provides more than one current path between two common points.

- Notice in the circuit below that between the two common electrical point A & B, there is I_{R1}, I_{R2}, I_{R3}, …. I_{Rn}.

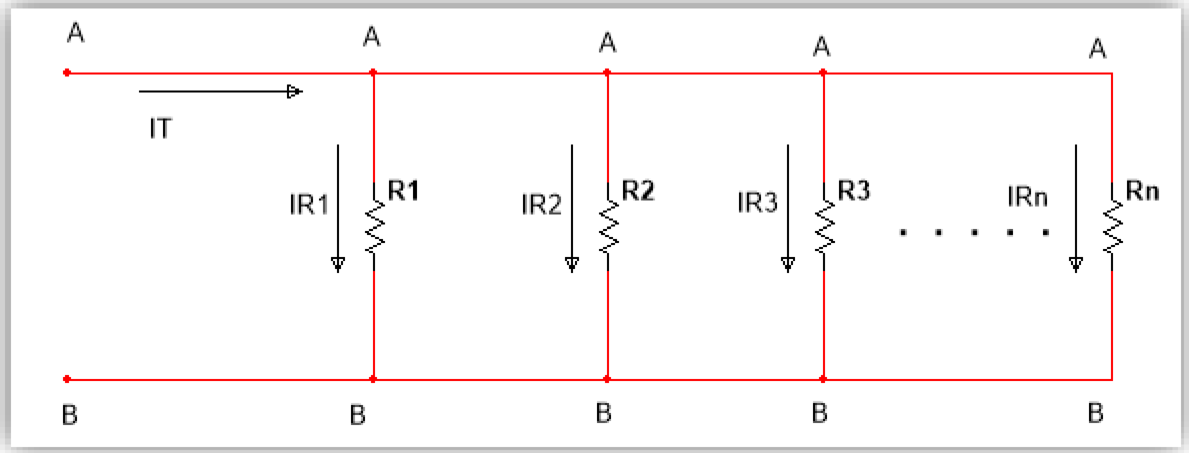


Parallel Circuit Characteristics

- Notice in the circuit below that between the two common electrical points A & B, there is I_{R1} , I_{R2} , I_{R3} , …. I_{Rn} .

- Notice that the current entering Node A which is electrical point A is called I_T and it is divided between the various parallel branches. Therefore:

$$I_T = I_{R1} + I_{R2} + I_{R3} + ... I_{Rn}$$

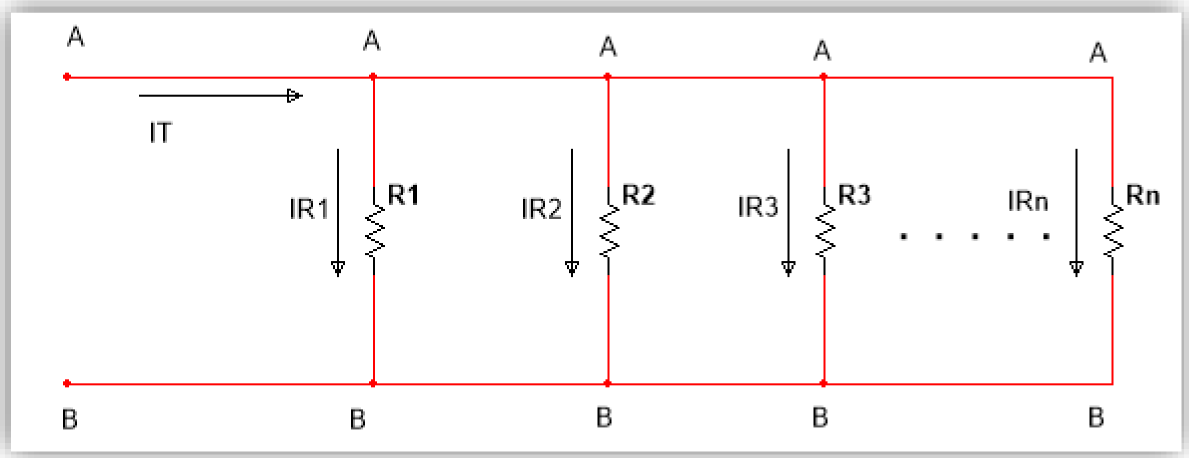


Parallel Circuit Characteristics

- The amount of current in each branch is based on the value of the resistance of that branch.

- The higher the resistance value, the lower the current through it. The lower the resistance value the higher the current through it.

- A resistance is called as such because it resists current flow. The current flowing through a resistance and the value of that resistance are inversely proportional to each other.



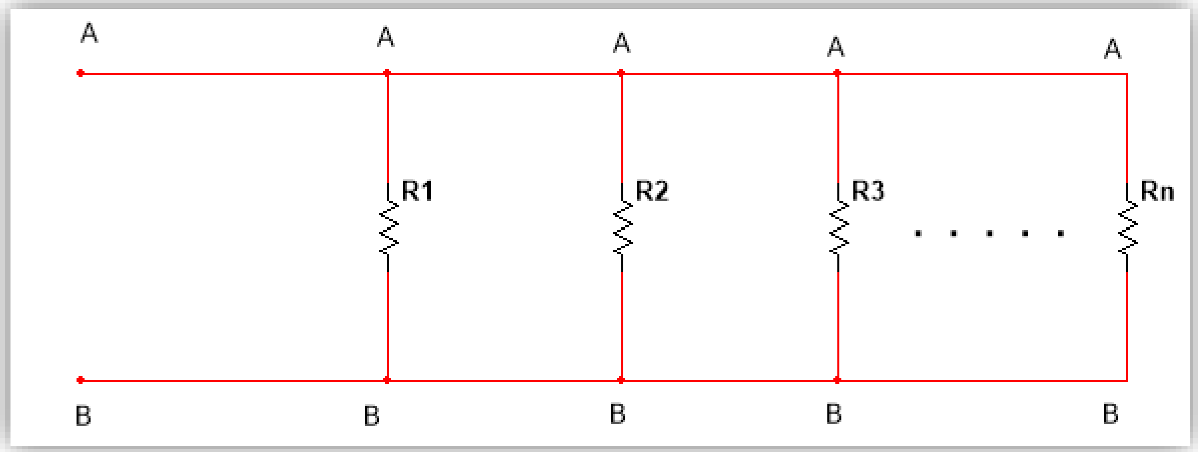
Parallel Circuit Total Resistance R_T

- The total resistance RT seen between point A & B is given by the following formula:

$$R_T = \frac{1}{\left(\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3} + \ldots . \frac{1}{Rn}\right)}$$

$$\text{If } R_1 = R_2 = R_3 = \ldots \ldots R_n = R, \text{ then: } R_T = \frac{R}{n}$$

- **The equivalent resistance of a parallel combination will always be lower than the lowest resistance of the parallel combination.**



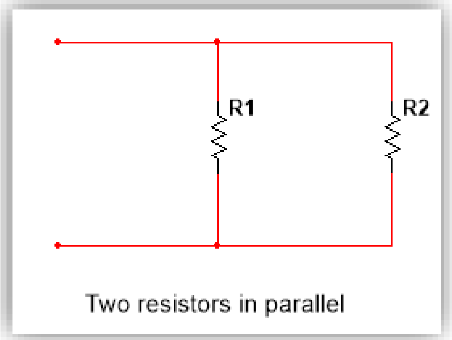
Two Resistor Parallel Circuit Total Resistance RT

- When only two resistors are in parallel, the reciprocal formula can still be applied, and the total resistance is given by:

$$R_T = \frac{1}{\left(\frac{1}{R1} + \frac{1}{R2}\right)}$$

- However, **only for two resistances in parallel**, the total resistance is also given by their product divided by their sum:

$$R_T = \frac{R_1 \times R_2}{(R_1 + R_2)}$$

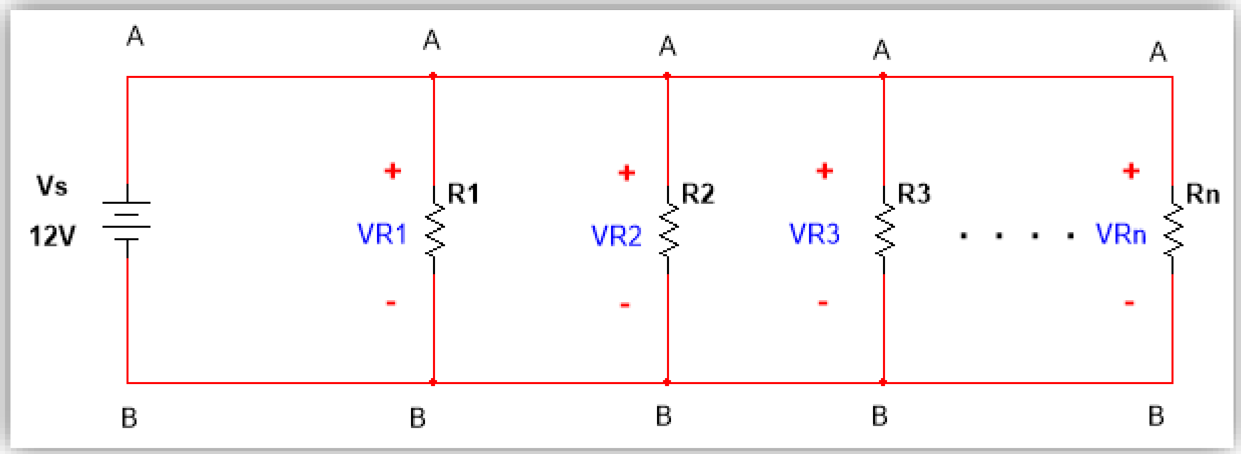


Voltages across Elements In Parallel Circuits

- The voltage definition is the potential difference between two points.

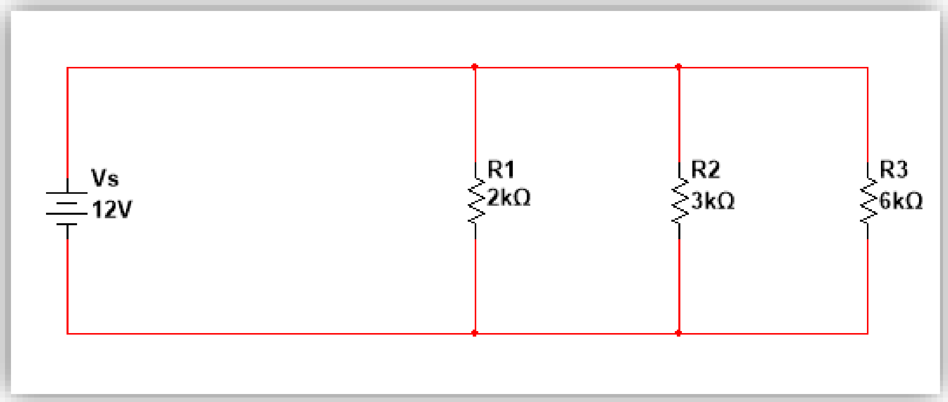
- All elements in the parallel circuits below fall between points A and B. Therefore, the voltage across each element would be the same as the voltage between points A and B.

$$V_{AB} = V_s = V_{R1} = V_{R2} = V_{R3} = \ldots V_{Rn}$$



Parallel Circuit Example

- For the following Circuit find:
  - a- Find the total resistance seen by the source
  - b- the voltages across R1, R2 & R3.
  - c- Find the currents through R1, R2 & R3
  - d- What is the total current supplied by the voltage source.
  - e - Find the power consumed by R1, R2 & R3.
  - f- Find the power delivered by the voltage source.

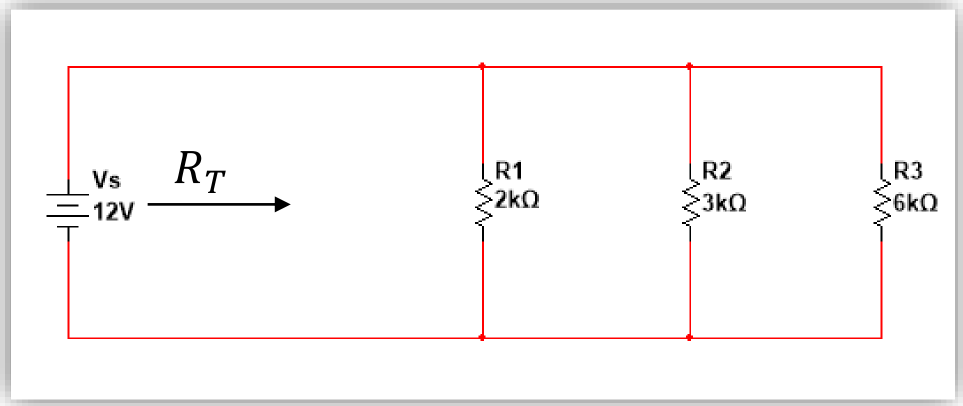


Parallel Circuit Example

- For the following Circuit find:
  - a- Find the total resistance seen by the source

- The total resistance is given by: $R_T = \frac{1}{(\frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3})} = \frac{1}{(\frac{1}{2K} + \frac{1}{3K} + \frac{1}{6K})} = 1K\Omega$

- This proves that the total resistance of 1 K is lower than the lowest resistance in the parallel branches (which is 2K).

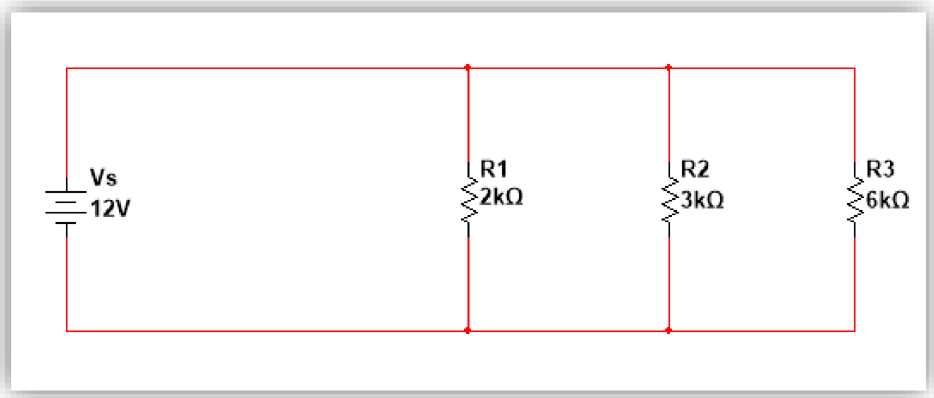


Parallel Circuit Example

- For the following Circuit find:
  - b- The voltages across R1, R2 & R3.

- Since all parallel branches have the same voltage across them, therefore:

$$V_s = V_{R1} = V_{R2} = V_{R3} = 12\text{V}$$



Parallel Circuit Example

- For the following Circuit find:
  - c- Find the currents through R1, R2 & R3

- Using Ohm's Law, The formula used to calculate the current through a resistance is equal to the voltage across the resistance divided by the resistance. It is given by the general equation:

$$I_{Rx} = \frac{V_{Rx}}{Rx}$$

- Where:
  - I_{Rx} is the current through Rx.
  - V_{Rx} is the voltage across resistance Rx.
  - Rx is the resistance.

Parallel Circuit Example

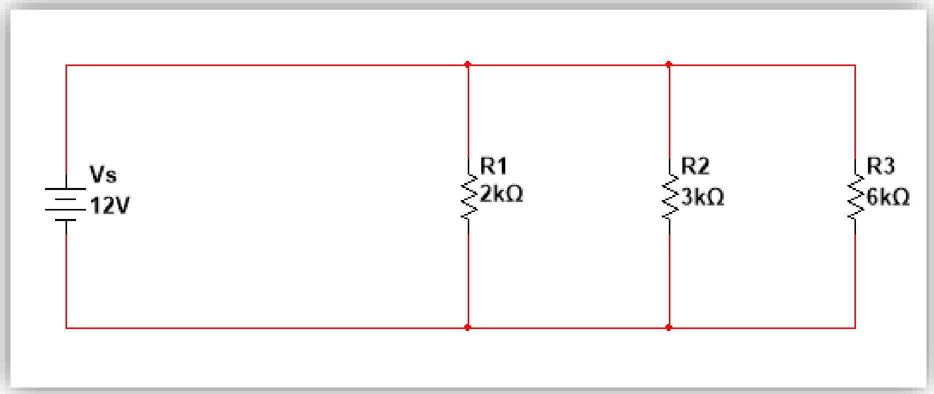
- For the following Circuit find:
  - c- Find the currents through R1, R2 & R3

- Therefore:

$$I_{R1} = \frac{V_{R1}}{R1} = \frac{12\ V}{2K} = 6\ \text{mA}$$

$$I_{R2} = \frac{V_{R2}}{R2} = \frac{12\ V}{3K} = 4\ \text{mA}$$

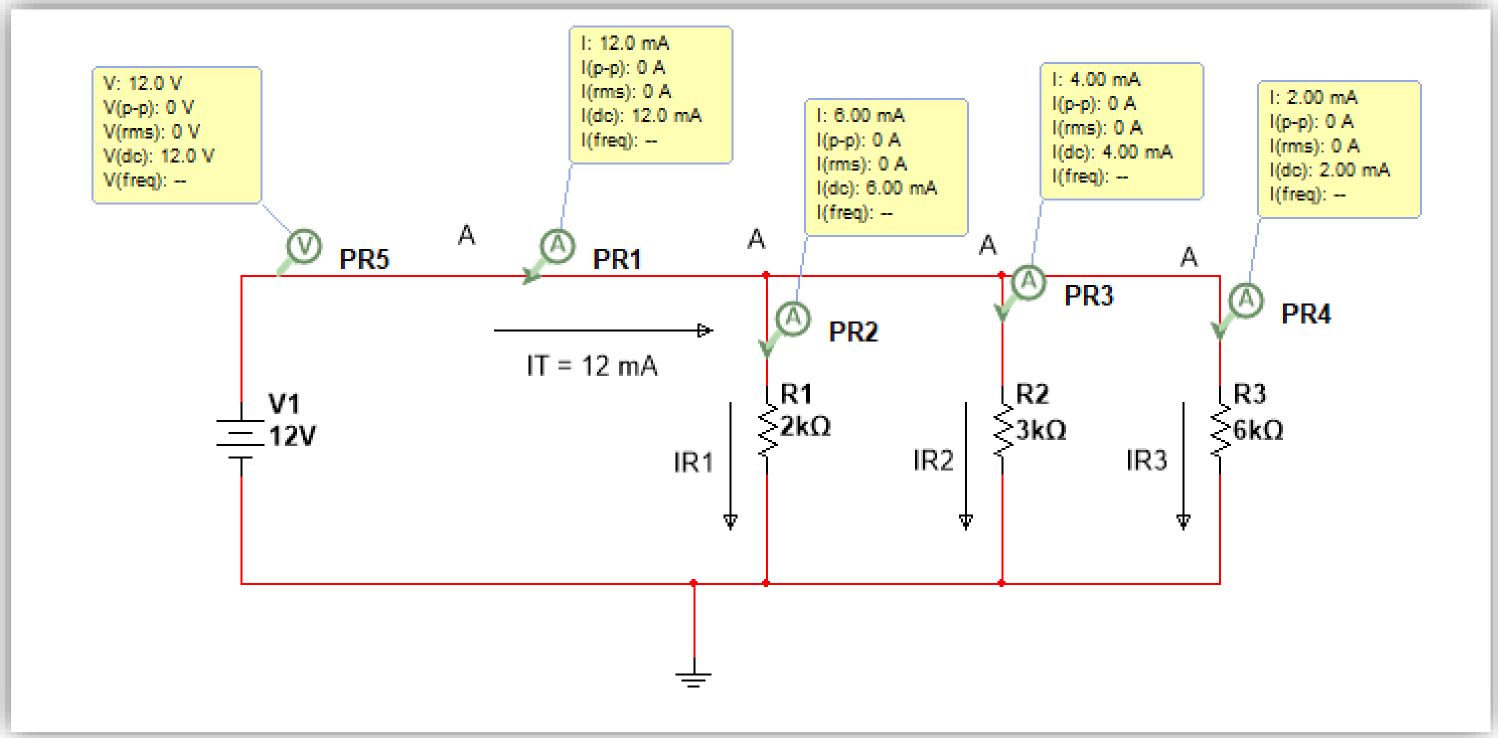
$$I_{R3} = \frac{V_{R3}}{R3} = \frac{12\ V}{6K} = 2\ \text{mA}$$



Parallel Circuit Example

- For the following Circuit find:
  - c- Find the currents through R1, R2 & R3

- Running the simulation using MultiSim yielded the following:



Parallel Circuit Example

- For the following Circuit find:

  d- What is the total current supplied by the voltage source.

- The total current sourced by the voltage source is calculated either by using ohm's law:

$$I_T = \frac{V_T}{RT} = \frac{12\ V}{1K} = 12\ \text{mA}$$

- It can also be found by simply adding the individual currents flowing through the individual resistors of the parallel combination:

$$I_T = I_{R1} + I_{R2} + I_{R3} = 6\ \text{mA} + 4\ \text{mA} + 2\ \text{mA} = 12\ \text{mA}$$

Parallel Circuit Example

- For the following Circuit find:
  - d- Find the power consumed by R1, R2 & R3.

- The Power consumed by a resistance is given by any of the following formulas:

$$P_R = V_R \times I_R$$

$$P_R = \frac{V_R^2}{R}$$

$$P_R = I_R^2 \times R$$

- Where:
  - P_R is the power consumed by R.
  - V_R is the voltage across R.
  - I_R is the current through R.

Parallel Circuit Example

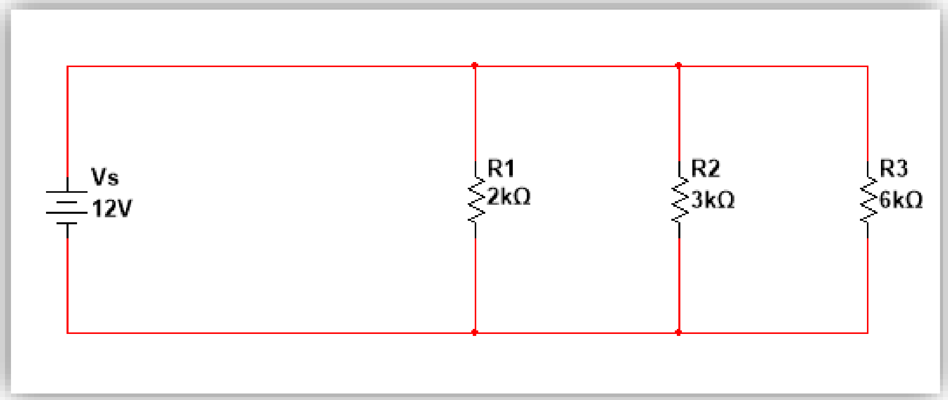
- For the following Circuit find:
  - d- Find the power consumed by R1, R2 & R3.

- The Power consumed by a resistance is given by any of the following formulas:
- Power consumed by R1 is given by:

P_{R1} = V_{R1} $\times$ I_{R1}= 12V × 6mA = 72 mW.

P_{R1} = $\frac{V_{R1}^2}{R1}$ = $\frac{12V^2}{2K\Omega}$ = $\frac{144}{2K\Omega}$ = 72 mW.

P_{R1} = I_{R1}^2 $\times$ *R1* = $(6mA)^2$ $\times$ 2KΩ = 36 µ × 2KΩ = 72 mW.



Parallel Circuit Example

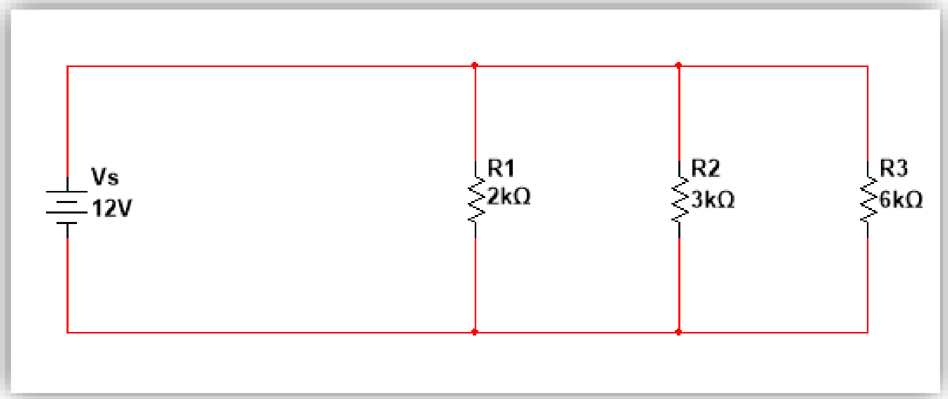
- For the following Circuit find:
  - d- Find the power consumed by R1, R2 & R3.

- The Power consumed by a resistance is given by any of the following formulas:
- Power consumed by R2 is given by:

$P_{R2} = V_{R2} \times I_{R2} = 12V \times 4mA = 48\ mW.$

$P_{R2} = \frac{V_{R2}^2}{R2} = \frac{12V^2}{3K\Omega} = \frac{144}{3K\Omega} = 48\ mW.$

$P_{R2} = I_{R2}^2 \times R2 = (4mA)^2 \times 3K\Omega = 16\ \mu \times 3K\Omega = 48\ mW.$



Parallel Circuit Example

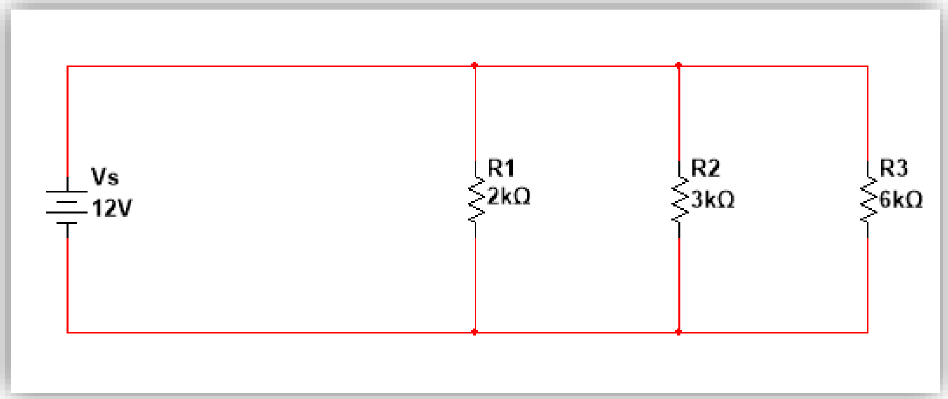
- For the following Circuit find:
  d- Find the power consumed by R1, R2 & R3.

- The Power consumed by a resistance is given by any of the following formulas:
- Power consumed by R3 is given by:

$$P_{R3} = V_{R3} \times I_{R3} = 12V \times 2mA = 24\ mW.$$

$$P_{R3} = \frac{V_{R3}^2}{R3} = \frac{12V^2}{6K\Omega} = \frac{144}{6K\Omega} = 24\ mW.$$

$$P_{R3} = I_{R3}^2 \times R3 = (2mA)^2 \times 6K\Omega = 24\ mW.$$



Parallel Circuit Example

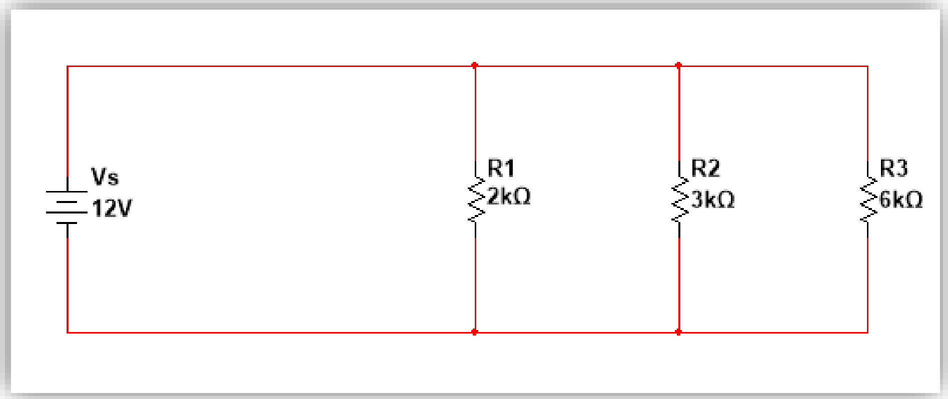
- For the following Circuit find:

  e- Find the power delivered by the voltage source.

  Power delivered by the voltage source is always equal to the sum of the powers consumed by the individual resistances. Therefore:

  Power Delivered = $P_{R1} + P_{R2} + P_{R3}$ = 72 mW + 48 mW + 24 mW = 144 mW which is the power delivered by the voltage source.

  Another way of finding the power delivered by the voltage source is obtained using the formula: $P_s = V_s \times I_s = V_s \times I_T$ = = 12V × 12 mA = 144 mW.

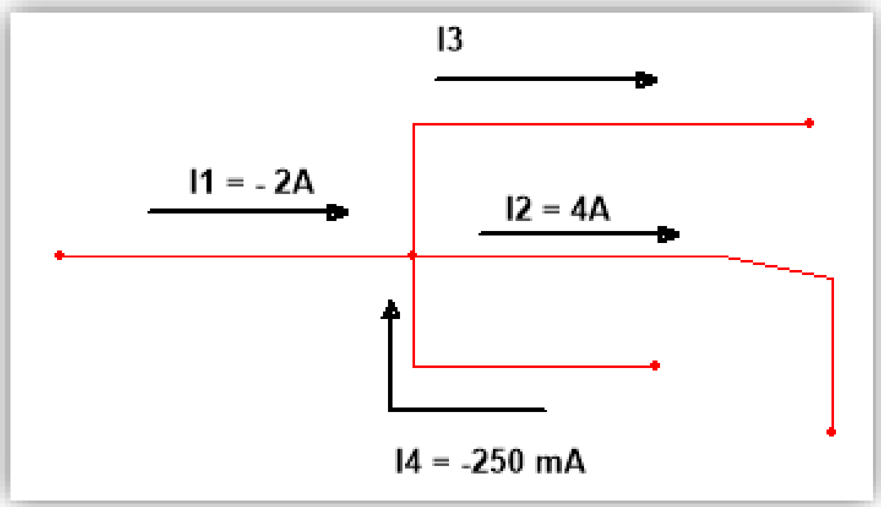


Kirchhoff’s Current Law, KCL

- Kirchhoff’s Current Law (KCL) Can be stated as one of the following:
- the algebraic sum of the currents entering a point (node) is equal to the sum of the currents leaving that same point (node).
- If all currents are entering a point (node), their sum will be equal to 0.
- If all currents are leaving a point (node), their sum will also be equal to 0.
- Where a Node is an electrical point connecting two or more elements

Kirchhoff's Current Law, KCL Example

- For the following circuit, what is the value of I_3?
- Based on KCL, the sum of all currents entering a node is equal to the sum of all currents leaving that node. Therefore:
  - $I_1 + I_4 = I_2 + I_3$
  - $I_3 = I_1 + I_4 - I_2$
  - $I_3 = -2A + (-250\,mA) - 4A = -6.25A$



Current Division Formula

- Current division only applies to a parallel combination of components.
- A total current I_T entering a node that contain n parallel resistances, will be divided between the resistances based on the following formula:
- $I_{Rx} = I_T \times \frac{R_T}{R_x}$, Where x = 1, 2, 3, ….n, where R_T is the total resistance of the parallel branch. Therefore:

  - $I_{R1} = I_T \times \frac{R_T}{R_1}$
  - $I_{R2} = I_T \times \frac{R_T}{R_2}$
  - $I_{R3} = I_T \times \frac{R_T}{R_3}$ …..
  - $I_{Rn} = I_T \times \frac{R_T}{R_n}$

Current Division Formula Example

- Find the values of I_{R1} , I_{R2} and I_{R3} for the following parallel combination using the current division formula and verify KCL.

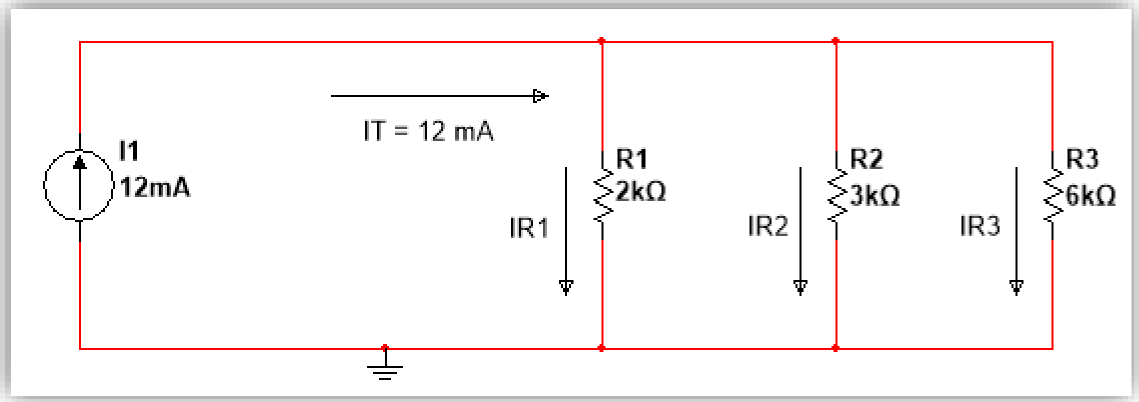
- $I_{Rx} = I_T \times \frac{R_T}{R_x}$, where x = 1, 2, 3, ….n, Therefore:

- $R_T = \frac{1}{(\frac{1}{R1}+\frac{1}{R2}+\frac{1}{R3})} = \frac{1}{(\frac{1}{2K}+\frac{1}{3K}+\frac{1}{6K})} = 1K\Omega$

- $I_{R1} = I_T \times \frac{R_T}{R_1} = 12 \times \frac{1K\Omega}{2K\Omega} = 6\ mA.$

- $I_{R2} = I_T \times \frac{R_T}{R_2} = 12 \times \frac{1K\Omega}{3K\Omega} = 4\ mA.$

- $I_{R3} = I_T \times \frac{R_T}{R_3} = 12 \times \frac{1K\Omega}{6K\Omega} = 2\ mA.$



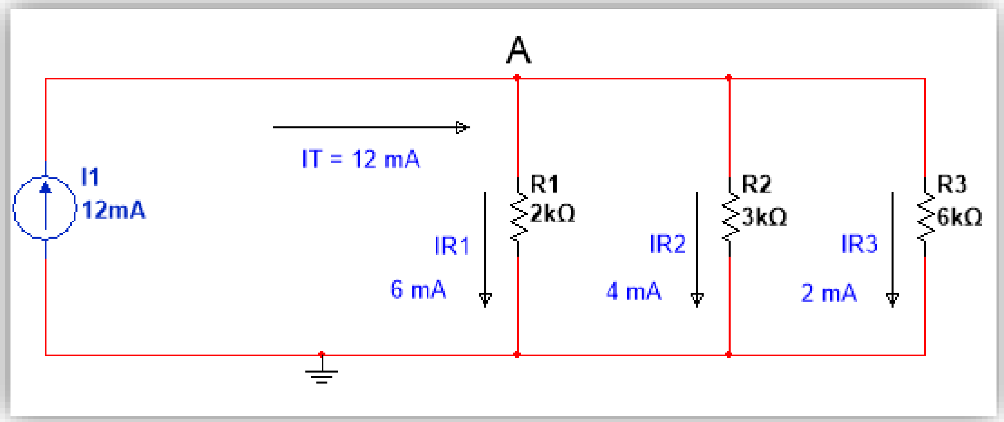
Current Division Formula Example

- Find the values of I_{R1} , I_{R2} and I_{R3} for the following parallel combination using the current division formula and verify KCL.

- Verifying KCL:
- KCL states that the sum of currents entering a node is equal to the sum leaving that same node. From the circuit shown:
- There is only one current entering Node A which is the total current and it is I_T and it is equal to 12 mA.
- The currents leaving node A are: I_{R1} = 6 mA, I_{R2} = 4 mA & I_{R3} = 2 mA.

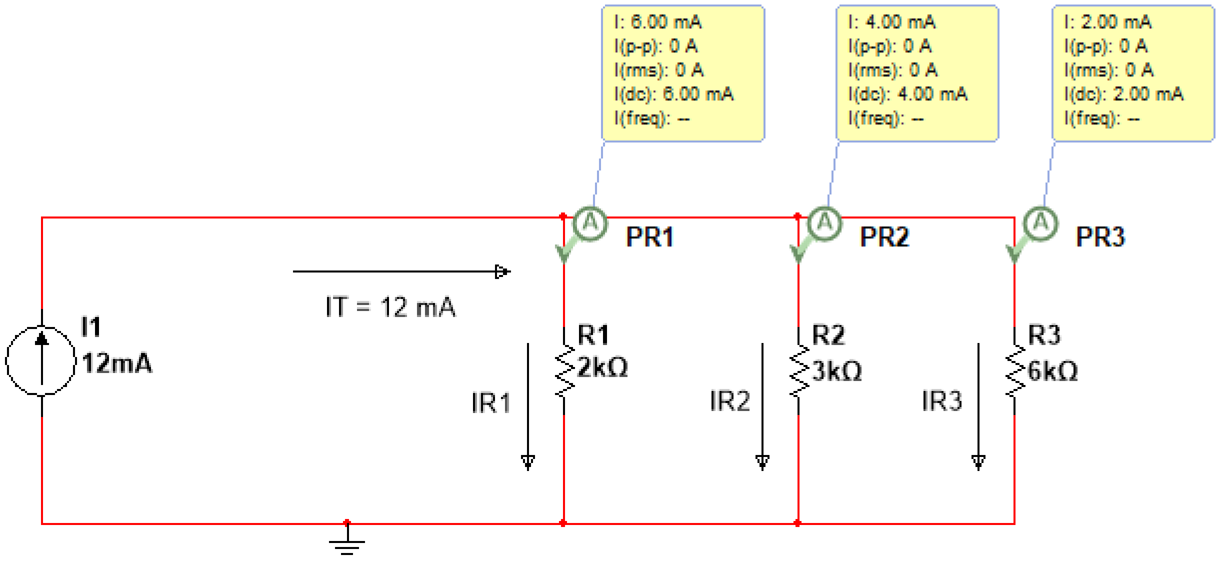
I_{R1} + I_{R2} + I_{R3} = 6 mA + 4 mA + 2 mA = 12 mA which is the value of I_T =12 mA

- This verifies KCL.



Current Division Formula Example

- Using MultiSim, the simulation for the parallel circuit yields:



This Concludes The Parallel Resistive Circuits PPT